ANALYTICAL STUDY OF A DEVICE FOR LOADING OF PET BOTTLES IN ROTARY CRUSHERS

Misiats V.[1], Skyba M.[2], Polishchuk O.[2], Rubanka M.[1], Misiats O.[2], Musial J.[3]

[1] Kyiv National University of Technology and Design, Ukraine
[2] Khmelnytskyi National University, Ukraine
[3] UTP - WIM – Bydgoszcz, Poland

Introduction

When grinding the waste of sheet polymeric materials, the rotary crusher has a fairly high productivity and uniformity of work. The main criterion for these indicators is the correct loading of waste into the working chamber, which is carried out manually by the operator.

However, the grinding of used bottles made of polyethylene terephthalate revealed a significant drawback of the process of loading them into the crusher. The containers getting into the working chamber are in contact with the movable blades of the rotor, are chaotically reflected from them and the process of grinding is accompanied by delays. The reason is the proportionality of the space between the knives and the size of the containers.

Research results

Rotary knife crushers [1, 2] concentrate the cutting of waste with knives, which are located both in the rotor and in the stator.

For the studies, a rotary crusher was used, the kinematic scheme and appearance of which are presented in Figs. 4.1 and 4.2 respectively.

The rotary crusher consists (Fig. 1) of the housing 5, the hopper 10, the tray 11, the rotor 6 with the knives 7 fixed thereto, the fixed knives 8, fixed on the housing, the variable calibrated grating 9, the size of the holes of which specifies the required dispersion of the crushed materials. The torque is transmitted from the motor shaft 1 to the rotor by a belt drive consisting of a pulley 3, a flywheel with a pulley 4 and a wedge belt 2.

The process of grinding the material begins with its loading in the window 10, from where it enters the rotor and its blades. The material is crushed by the interaction with the rotor blades and the fixed stator blades. After that, the material is sifted through the grid and falls into the hopper of the finished product. This type of shredder is quite widely used in the chemical, pulp and paper, food, construction and light industries [3].
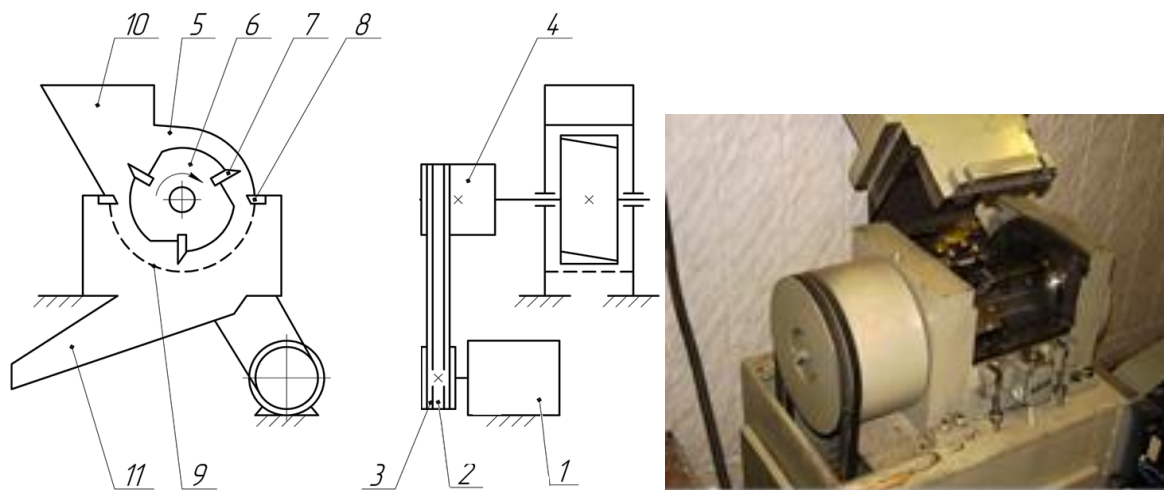
Fig. 1. Rotary knife crusher: a - kinematic scheme; appearance; 1 - electric motor; 2 - wedge belts; 3 - pulley; 4 - flywheel with pulley; 5 - grinding chamber; 6 - rotor; 7 - rotor blades; 8 - fixed knives; 9 - calibrated mesh; 10 - loading window; 11 – tray

We carried out preliminary experiments [4], during which the time of grinding of bottles with a capacity of 2 liters was determined with successive continuous loading. Experiments have shown that when loading bottles that are cut, along the three parts, the crushing performance is 5… 6 times higher than when loading a whole container.

As a result of the experiment, it was concluded that it is advisable to equip the crusher with a roller for deforming and pre-cutting the used bottles, which should be installed at the entrance to the working chamber.

The scheme of the device is presented in Fig. 2. It consists of two rolls, which are profiled in such a way that in the middle part a reliable grip of the neck of the containers is possible due to their sufficiently large thickness and rigidity.

Two pairs of disc blades 3 and 4 are secured to the end surfaces of the rolls, which, after deformation of the thin-walled part of the container, are cut into three sections by rolls. When feeding the cut pieces to the working chamber of the crusher, the intensity of the further grinding process will increase significantly.

The proposed design of the roller device requires analytical study of its performance and experimental verification.

Consider the conditions of material capture (PET bottles) by rollers.

Due to the progressive movement of the material and the inevitable roughness of the contact surfaces of the bottles and shafts under certain conditions, the material is captured by the rolls and drawn into the space between the rolls.

The center angle between the line joining the centers of the shafts and the radius passing through the point of contact of the contacting surface of the material with the shafts is called the angle of capture. The normal and tangential forces applied to the material through its contact surfaces act on the shaft.
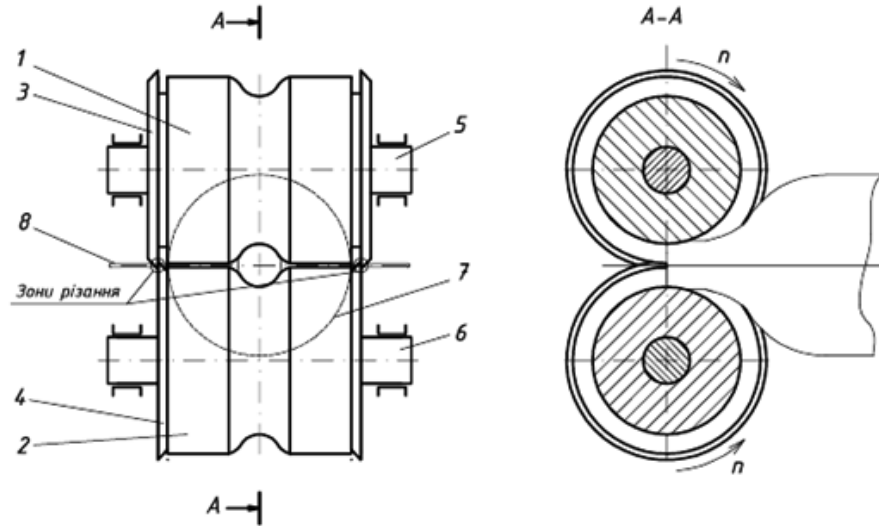

Figure 2. Scheme of the roller device for deformation and preliminary cutting of used thin-walled containers when loaded into a rotary crusher: 1, 2 - profiled rolls; 3, 4 - pairs of disc knives; 5, 6 - shafts; 7 - capacity in the deformed state; 8 - capacity after compression in between the roll gap

Determine the condition of capture of the material by the rolls at the initial moment of transportation. The material in contact with the rolls is initially affected by the forces of normal pressure N and friction, F_1, F_2 (Fig. 3a).
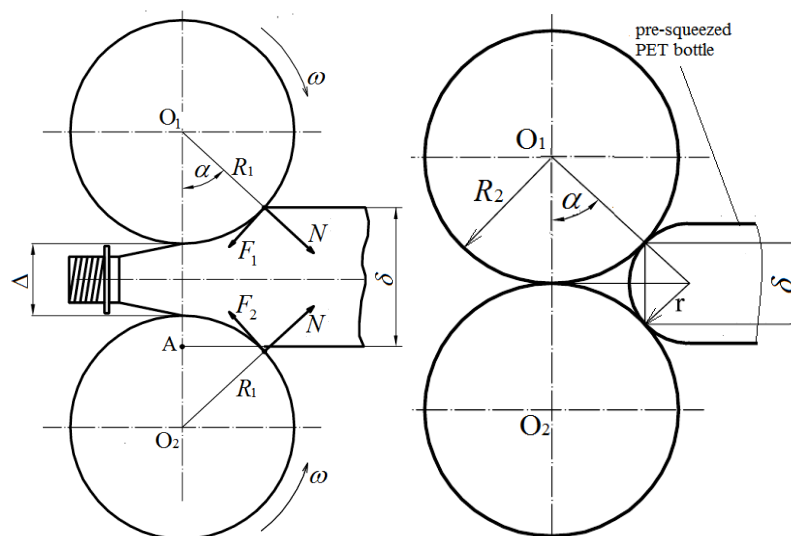

Fig. 3. Schemes of interaction of PET bottles with rolls: a - in the central part; b - in the compression zones

We design the vectors of active forces on the X-axis and obtain the condition of material trapping by rolls:

$$(F_1 + F_2)\cos\alpha > 2N\sin\alpha, \tag{1}$$

where α - the angle of capture of the material by rolls; N - the normal force of the rolls pressure on the material; F_1, F_2 friction forces between the rolls and the material.

Provided that the surfaces of the rolls are the same and that they are in contact with the material $F_1 = F_2 = F_{mep}$.

$$F_{mep} = N \cdot f, \tag{2}$$

where f - the coefficient of friction of the material on the surface of the rolls. Taking into account (2), from (1) we obtain the condition of material trapping by rolls at the initial moment of transportation:

$$tg\alpha < f. \tag{3}$$

According to the scheme in Fig. 3a we can write the equation:

$$R_1 + \frac{\Delta}{2} = R_1\cos\alpha + \frac{\delta}{2}.$$

Hence

$$\Delta_{min} = \delta - 2R_1(1 - \cos\alpha). \tag{4}$$

From the scheme in Fig. 3a shows that

$$AO_2 = R_1 - \frac{\delta - \Delta}{2}, \text{ or } AO_2 = R_1 \cdot \cos\alpha. \tag{5}$$

From (4) and (5) we get:

$$\cos\alpha = 1 - \frac{\delta - \Delta}{2R_1}. \tag{6}$$

Let us imagine $tg\alpha$ in a:

$$tg\alpha = \frac{\sqrt{1 - \cos^2\alpha}}{\cos\alpha}. \tag{7}$$

Given expression (6), formula (7) takes the form:

$$tg\alpha = \frac{\sqrt{1-\left(1-\dfrac{\delta-\Delta}{2R_1}\right)^2}}{1-\dfrac{\delta-\Delta}{2R_1}}. \tag{8}$$

We introduce the notation $\dfrac{\delta-\Delta}{2R}=x$ and rewrite (8) with (3) in the form:

$$f = \sqrt{1-(1-x)^2}\Big/(1-x). \tag{9}$$

Perform the algebraic transformations of expression (9) and obtain the equation:

$$\left(1+f^2\right)x^2 - 2\left(1+f^2\right)x + f^2 = 0. \tag{10}$$

The solution of the obtained quadratic equation (10) is:

$$x_{1,2} = \frac{1}{2}\left(1\pm\frac{1}{\sqrt{\left(1+f^2\right)}}\right). \tag{11}$$

We are satisfied with the solution (11), in which the points of contact of the material are in the middle part of the rolls (as in Fig. 3a), ie with the sign "minus". Substitute in (11) instead of its value and obtain after transformations:

$$R_1 = \frac{\delta-\Delta}{\left(1-\dfrac{1}{\sqrt{\left(1+f^2\right)}}\right)}. \tag{12}$$

The obtained formula (12) allows us to determine the minimum radius of the rolls, which provides the condition of trapping of the material, with known values of its initial thickness δ, clearance between the rollers Δ and the coefficient of friction f.

In the case of initial contact of the rolls and the bottle, which is pre-compressed to a thickness $2r$ (Fig. 3b) the value can be found according to the scheme in Fig. 3b and is expressed by the formula:

$$\delta = 2r \cdot R_2 / (r + R_2).$$ (13)

Because $\Delta \ll \delta$, for simplicity we consider $\Delta = 0$, then formula (12), taking into account (13), takes the form:

$$R_2 = \frac{2r}{\left(1 - \dfrac{1}{\sqrt{(1 + f^2)}}\right)} - r.$$ (14)

The coefficient of friction of polyethylene terephthalate on steel depends on the condition of the contact surfaces and matters $f = 0{,}5 \ldots 0{,}7$.

In Fig. 4 presents a graph of the minimum required radius of the rolls to capture them PET bottles from r and the friction coefficient obtained by formula (14).

The graph shows that the capture of even almost fully compressed bottles, smooth rolls (when $r_{пл} = 1$ мм) guaranteed possible with $R > 20$ мм.

Further studies will need to perform mathematical modeling of the process of cutting compressed bottles with disc blades and to determine the design and energy parameters of the entire device.
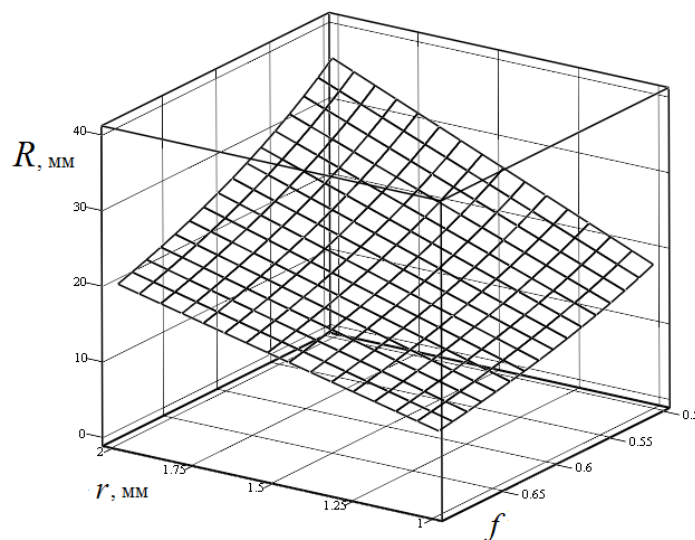

Fig. 4. Graph of the dependence of the minimum required radius of the rolls to capture their PET bottles from r and the coefficient of friction f

Analytical study of disk scissors

Disc blades in the roller device are used to cut pre-compressed rolls of bottles into three longitudinal parts. To obtain high-quality cutting, the disc blades are installed with radial overlap Δ.

The scheme of cutting material with disk knives is presented in Fig. 5. As can be seen from the diagram (Fig. 5), knives of the same radius $R_{_H}$ have overlapping magnitudes $\Delta_{_H}$, whereby a steam is formed. The material is thick h (when cutting PET bottles h is equal to twice the thickness of the walls of the bottle) is fed to the knives and in the area of the ABC (Fig. 5) is its cut.

Because the thickness of the material is much smaller than the knife radius ($h \ll R_{_H}$),then to simplify the arc of the BC and AC (Fig. 5) we replace the corresponding chords.

The angle of capture of the material with knives β (the angle of inclination of the cutting chord of the AC (see Fig. 5)) depends on the radius $R_{_H}$, thickness h and blade overlap values $\Delta_{_H}$ and can be found from the ratio of the segments of the lengths of the DC and AC (see Fig. 5):

From the scheme in Fig. 5 shows that:
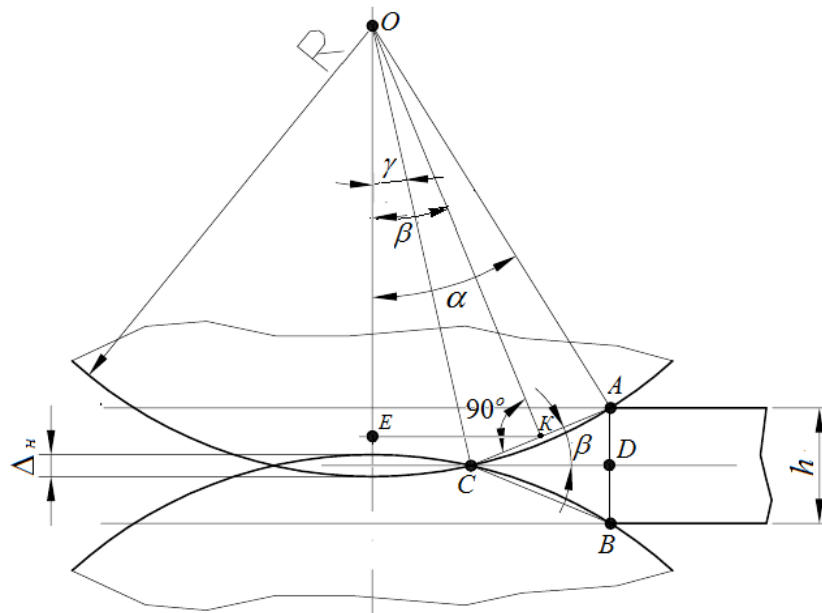
$$tg\,\beta = h/2DC .\tag{15}$$

Fig. 5. Scheme of cutting material with scissors

Disc blade overlap angle γ (fig. 5) can be defined from the expression:

$$\cos \gamma = \left(R_{_H} - \frac{\Delta_{_H}}{2} \right) \Big/ R_{_H}, \quad \text{or} \quad \cos \gamma = 1 - \frac{\Delta_{_H}}{2R_{_H}}. \qquad (16)$$

Chord angle of cutting AC - α (fig. 5), which can be defined from the expression:

$$\cos \alpha = 1 - \left(h + \Delta_{_H} \right) \Big/ 2R_{_H}. \qquad (17)$$

If expressions are known for α and γ, then the length of the segment DC, which is included in formula (15) can be found by means of the trigonometric relation (Fig. 5):

$$DC = R_{_H} \sin(\alpha) - R_{_H} \sin(\gamma). \qquad (18)$$

Let us represent (18) as:

$$DC = R_{_H} \sqrt{1 - \cos^2 \alpha} - R_{_H} \sqrt{1 - \cos^2 \gamma}. \qquad (19)$$

Substitute in (19) the expressions (16), (17) and obtain:

$$DC = R_{_H} \left(\sqrt{1 - \left(1 - \frac{h + \Delta_{_H}}{2R_{_H}} \right)^2} - \sqrt{1 - \left(1 - \frac{\Delta_{_H}}{2R_{_H}} \right)^2} \right). \qquad (20)$$

Substitute in (15) the expression (20) for DC and obtain:

$$tg\beta = \frac{h}{2R_{_H} \left(\sqrt{1 - \left(1 - \frac{h + \Delta_{_H}}{2R_{_H}} \right)^2} - \sqrt{1 - \left(1 - \frac{\Delta_{_H}}{2R_{_H}} \right)^2} \right)}. \qquad (21)$$

In Fig. 6 shows a plot of the values $tg\beta$ from the knife radius $R_{_H}$ and the magnitude of their overlap $\Delta_{_H}$, which is obtained from the formula (21) (at a material thickness $h = 1$ mm).

From the graph in Fig. 6. it is visible that at small diameters of disk knives and the big value of their overlapping $\Delta_{_H}$ material capture may not occur.
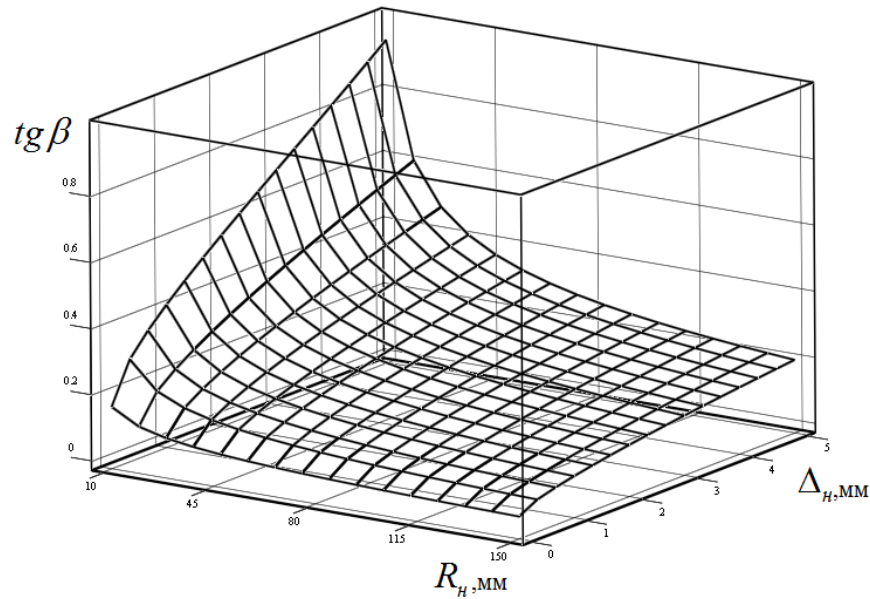
Fig. 6. Graph of value dependence $tg\beta$ **from the knife radius** $R_{_H}$ **and the magnitude of their overlap** $\Delta_{_H}$ **(at material thickness** $h=1$ **mm)**

At the thickness of the material $h=0,6$ mm (double the thickness of the bottle wall), the value of the knife overlap $\Delta_{_H}=2$ mm and their radius $R_{_H}=52$ mm, $tg\beta$ will obtain:

$$tg\beta = \frac{0,6}{104\left(\sqrt{1-\left(1-\dfrac{0,6+2}{104}\right)^2}-\sqrt{1-\left(1-\dfrac{2}{104}\right)^2}\right)} = 0,24.$$

Therefore, the value of the friction coefficient of PET $f_{mep}=0,6$ the condition of grabbing the material with knives will be fulfilled.

Determination of the cutting power of disc shears

Because the thickness of the material is much smaller than the knife radius ($h \ll R_{_H}$), then to simplify the arc BC and AC (fig. 5) we can replace the corresponding chords. In this case, the process of cutting with circular knives will be similar to the process of cutting straight slopes with straight knives (Fig. 7).
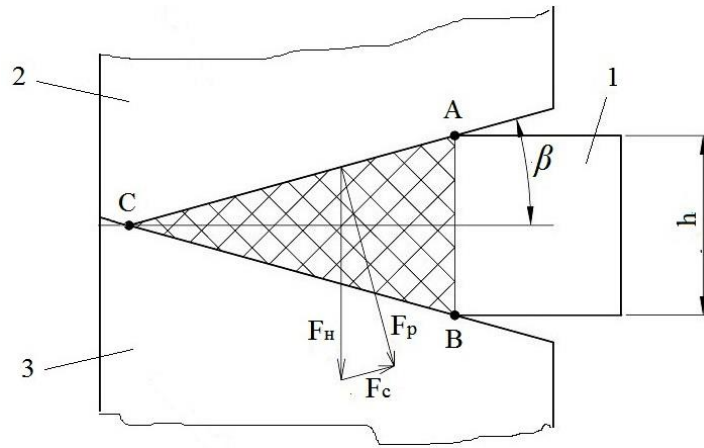
**Fig. 7. Scheme of cutting with inclined straight knives: 1 - cutting material;
2, 3 - knives**

In such a scheme of interaction of the cutting tool with the material, the maximum technological cutting force will be determined from the condition of the shear strength [5]:

$$F_{\text{н}} = \sigma_{\text{зр}} \cdot S_{\text{зр}}, \tag{22}$$

where $S_{\text{зр}}$ - the area was cut; $\sigma_{\text{зр}}$ - the ultimate strength of the material at the cut.

From the scheme in Fig. 7 the material cutoff will be:

$$S_{\text{зр}} = h^2 / tg\beta. \tag{23}$$

Then given (23), (22) takes the form:

$$F_{\text{н}} = \sigma_{\text{зр}} h^2 / tg\beta. \tag{24}$$

The tensile strength of the material at the cut, corresponding to the breaking stress $\sigma_{\text{зр}} = \sigma_p$, can be determined experimentally for each test material separately.

The moment of resistance M_{on} corresponds to the time of cutting M_p, which is determined by the formula:

$$M_p = F_{\text{н}} \cdot R_{\text{н}}. \tag{25}$$

Substituting (24) into (25) and taking into account (21) we obtain a formula for determining the moment of resistance of a rotary knife crusher, which has the following form:

~ 243 ~

$$M_{on} = \sigma_p h R_{_H}{}^2 \left(\sqrt{1 - \left(1 - \frac{h + \Delta_{_H}}{2R_{_H}}\right)^2} - \sqrt{1 - \left(1 - \frac{\Delta_{_H}}{2R_{_H}}\right)^2} \right). \qquad (26)$$

Power consumed by cutting two disc blades:

$$N_{pi3} = 2M_{on} \cdot \omega. \qquad (27)$$

Summary

Thus, we have obtained the main dependencies that can be used in the calculation of the structural and technological parameters of the device for pre-cutting the container of polyethylene terephthalate when loaded into rotary crushers.

References

1. Rubanka M.M. & Misiats V.P. (2016). Eksperymentalni doslidzhennia dynamiky rotornoi drobarky dlia pererobky vidkhodiv lehkoi promyslovosti [Experimental studies of the dynamics of a rotary crusher for the processing of waste of light industry]. Visnyk Kyivskoho natsionalnoho universytetu tekhnolohii ta dyzainu – Bulletin of the Kyiv National University of Technologies and Design, 1 (94), 27-36 [in Ukrainian].

2. Misiats V.P. & Rubanka M.M. (2014). Eksperymentalna ustanovka dlia doslidzhennia dynamichnykh kharakterystyk rotornoi nozhovoi drobarky [The experimental fluidizer research of dynamic characteristics of rotary knife crusher]. Pratsi Odeskoho politekhnichnoho universytetu – Odes'kyi Politechnichnyi Universytet. Pratsi, 1(43), 78-82 [in Ukrainian].

3. Rubanka M.M. & Misiats V.P. (2015). Vidkhody lehkoi promyslovosti, sposoby pererobky ta oblasti podalshoho vykorystannia [Waste of light industry, types of recycling and the follow-use]. Visnyk Kyivskoho natsionalnoho universytetu tekhnolohii ta dyzainu – Bulletin of the Kyiv National University of Technologies and Design, 4 (88), 34-39 [in Ukrainian].

4. Misiats V.P. & Misiats, O.V. (2018). Udoskonalennia rotornoi drobarky dlia podribnennia vykorystanoi tary z polietylentereftalatu [Improvement of the rotor crusher for grinding of used polyethylene terephthalate container]. Proceedings from ETSEP '18: Mizhnarodna naukovo-tekhnichna konferentsiia «Inzheneriia ta tekhnolohii: nauka, osvita, vyrobnytstvo» - International Scientific and Technical Conference «Engineering and Technology: Science, Education, Production». (pp. 168-170). Lutsk : LNTU [in Ukrainian].

5. Misiats V.P., Misiats O.V., Skyba M.Ye., Zozulia P.F. & Polishchuk A.O. (2018). Matematychne modeliuvannia prystroiu dlia poperednoho rozrizannia tary z polietylentereflatalu pid chas zavantazhennia v rotorni drobarky [Mathematical modeling of a device for pre-cutting the container from polyethylene terephthalate during loading into rotary crushers]. Visnyk Khmelnytskoho natsionalnoho universytetu - Herald of Khmelnytskyi national university, Vol. 2, 6 (267), 34-38 [in Ukrainian].