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CONDITION OF ORTHOGONALITY OF PROBABILISTIC MEASURES CORRESPONDING TO GAUSSIAN GENERALIZED RANDOM PROCESSES WITH INDEPENDENT VALUES

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Statistical analysis systems subject to random perturbations action substantially uses information about the relationship between probability measures in certain statistical structures [1] corresponding to the random functions describing perturbation mentioned [2]. In particular, the Gaussian functions are possible only their equivalence or orthogonality (a singularity). In the latter case, the aforementioned measures comply with incompatible observable events that makes it possible to uniquely identify the current stochastic perturbation.

In this paper we present the orthogonality condition of probabilistic measures corresponding to the generalized Gaussian random processes with independent values in terms of the covariance functions of these processes. Under the generalized random process$\xi = \xi(\varphi)$ we mean the set of real random variables$\left\{\xi_\varphi\right\}$ indexed by the elements $\varphi$of a linear space $\mathcal{A}$ offunctions on the real line R, which introduced a topology or convergence. It is supposed that the correspondence $\varphi \to \xi_\varphi$is linear with probability 1, and continuous. The latter means that for $\varphi_n \xrightarrow[n\to\infty]{} \varphi$(in the sense of convergence in $\mathcal{A}$) we have the convergence $\xi(\varphi_n) \to \xi(\varphi)$(in one or another probabilistic sense).Here$\mathcal{A}$ is the set $D((0,1))$of infinitely differentiable functions on having compact carriers in the interval (0,1). (For the convergence of sequences in such spaces seefor example [3]). All finite-dimensional distributions of these processes assumed to be normal (Gaussian). Independence of process valuesmeans that the random variables $\xi(\varphi)$, $\xi(\psi)$are independent, if it's supports do not have a common interior points. Such processes play an important role, for example, in the theory of stochastic differential equations A number of properties of these processes are given in [2, 3].Next E is a symbol of expectation. According to [3], the functional covariance
$B(\varphi,\psi) \stackrel{\text{def}}{=} E\xi(\varphi)\xi(\psi) - E\xi(\varphi)E\xi(\psi)$,
of the process is represented as

$$B(\varphi,\psi) = \int \sum_{j,k\geq 0} R_{jk}(x)\varphi^{(j)}(x)\psi^{(k)}(x)dx, \qquad (1)$$

where $\int \ldots = \int_{-\infty}^{\infty} \ldots$, R_{jk} are continuous functions, the number of terms is finite. Further (without loss of generality) the order of the differential form in (1), i.e. value $max_{j,k}\{j+k\}$ is considered an even number.

The main result of the work is the following statement

Theorem. Let $(\Omega, \Sigma, P_1, P_2)$ be a statistical structure. Suppose that, with respect to probability measures $P_i, i = 1,2$, the process ξ (φ), φ∈*D*((0,1)) is Gaussian with zero mean values and the corresponding P_i, covariance functions $B_i(\varphi, \psi), i = 1,2$ admitting (according to (1))

$$B_1(\varphi, \psi) = \sum_{\substack{k,j=0, \\ k+j \le 2N_i}}^{2N_i} \int R_{kj}(x)\varphi^{(k)}(x)\psi^{(j)}(x)dx,$$

$$B_2(\varphi, \psi) = \sum_{\substack{k,j=0, \\ k+j \le 2N_i}}^{2N_2} \int Q_{kj}(x)\varphi^{(k)}(x)\psi^{(j)}(x)dx.$$

Then if $N_1 \neq N_2$, then the measures P_1 and P_2 are orthogonal. If $N_1 = N_2 = N$, then P_1 and P_2 are orthogonal when the inequality takes place

$$\sum_{i=0}^{2N}(-1)^{N-i}\int_0^1 R_{i,2N-i}(x)dx \neq \sum_{i=0}^{2N}(-1)^{N-i}\int_0^1 Q_{i,2N-i}(x)dx.$$

Note that in the case when the functions $R_{kj}(x), Q_{kj}(x)$ are constant values, the above theoremgives also necessary conditions for the orthogonality of the measures P_1 and P_2. This follows from the results of [4,5].

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