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ALGORITHM FOR FINDING ROTATION ANGLES A AND B, RESPECTIVELY, FOR STATIONARY AND MOVING FLAT GEOMETRIC OBJECTS WHEN CONSTRUCTING DENSE STACKS OF THESE OBJECTS

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Introduction. Rational and economic spending of material and energy resources, as well as protection of the environment from pollution have always been and are priority directions in the development of Ukraine. And for this, it is necessary to reduce the amount of waste. Thus, materials make up more than 80% of the cost of shoes, and the technological features of shoe production lead to the fact that only the waste of cutting shoe materials makes up more than 20%, so the importance of rational use of materials is obvious.

Aim. Development of algorithms and a software product for the design of dense lattice stacks for two types.

Materials and methods. The object of research is the technological process of cutting rectangular materials into haberdashery details. The subject of the study is the automated design of rational schemes for cutting rectangular materials into haberdashery details. The research methods are based on the basic provisions of the haberdashery production technology, mathematical modeling, the theory of lattice laying and the methods of computer graphics, computational mathematics and analytical geometry.

Results and discussion. The technological formulation of the problem of designing dense lattice stacks for two types of flat geometric objects with different configurations of external contours is formulated as follows: find the densest stacking in a parallelogram for two types of flat geometric objects S_1 and S_2 with different configurations of external contours, that is, find a parallelogram of minimum area (Fig. 1).

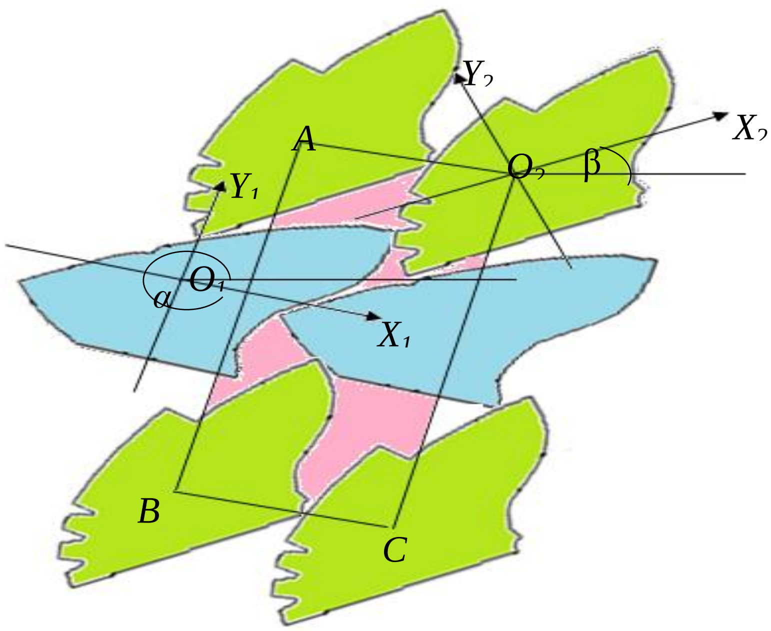

Fig. 1. Dense stacks for two flat geometric objects with different outer contour configurations

Let's construct a hodograph of a vector-function of dense placement (HVFDP)[1] for fixed and moving flat geometric objects with themselves. Let the immovable flat geometric object S1 be a polygon $G^n\{X_i^n, Y_i^n\}, i = 1,2...n$ (Fig. 2), and let the moving flat geometric object S2 be a polygon $G^r\{X_j^r, Y_j^r\}, j = 1,2...m.$ (Fig. 3).

Let's choose an arbitrary point $A(Xa,Ya)$ on the outer contour of the HVFDP $G^n\{X_i^n, Y_i^n\}$ (Fig. 2). Now, for each vertex on the HVFDP $G^n\{X_i^n, Y_i^n\}$, we will find the corresponding points on the HVFDP $G^r\{X_j^r, Y_j^r\}$, for which segments $A0=0Bi=R$, $i=1,2...q$.

This problem is solved as follows:

-we choose a point $A(Xa, Ya)$ on the HVFDP $G^n\{X_i^n, Y_i^n\}$ and find it $R = \sqrt{(Xa)^2 + (Ya)^2}$;

- we find the angle of rotation α from the condition $\angle\alpha = \angle AOX$ that (Fig. 2). Then we will have $Sin\ \alpha = {Ya}/{R}$; $Cos\ \alpha = {X}/{R}$;

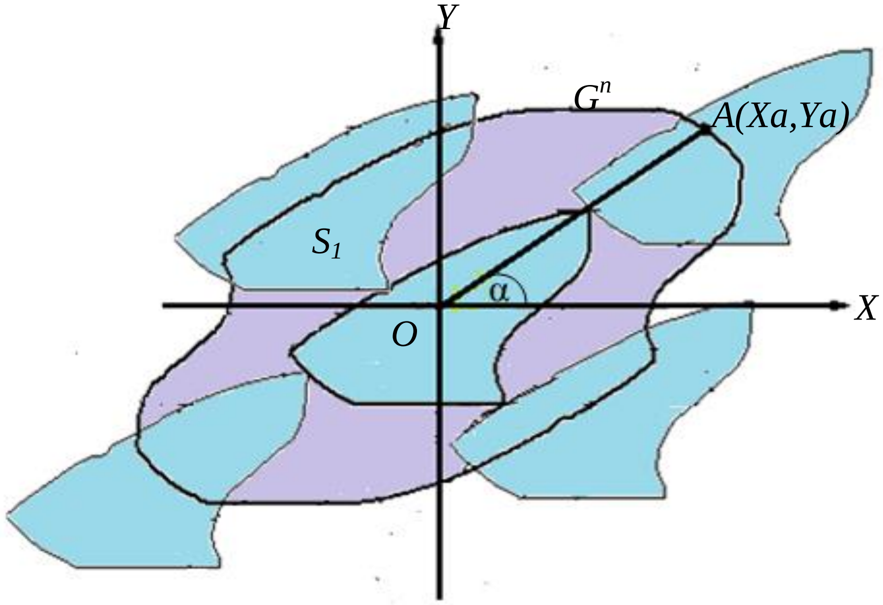

Fig. 2. HVFDP for flat geometric object S_1

- we find on HVFDP $G^r\{X_j^r, Y_j^r\}$ the points B_1, B_2, B_3, B_4 (Fig. 3) for which $|OA| = |OB_1| = |OB_2| = |OB_3| = |OB_4|$. That is, it is necessary to find the points of intersection of a circle with a radius *R* with the center at the origin of coordinates and the outer contour of the HVFDP for the moving flat geometric object S_2 (Fig. 3);

Let the sought points have the following coordinates $B_1(Xb_1, Yb_1)$, $B_2(Xb_2, Yb_2)$, $B_3(Xb_3, Yb_3)$;

- we find the coordinates of the points of intersection of the HVFDP and the circle by solving the following system:

$$\begin{cases} A_j x + B_j y + C_j = 0 \\ x^2 + y^2 = R^2 \\ T_1 \le x \le T_2; \ \ Q_1 \le x \le Q_2 \end{cases} \quad (1)$$

where

$$i = 1..m-1;$$

$$A_j = Y^r_{j+1} - Y^r_j; \quad B_j = X^r_j - X^r_{j+1}; \; C_j = X^r_{j+1} Y^r_j - Y^r_{j+1} X^r_j; \tag{2}$$

$$T_1 = \min\{X^r_j, X^r_{j+1}\}; T_2 = \max\{X^r_j, X^r_{j+1}\}; Q_1 = \min\{Y^r_j, Y^r_{j+1}\}; Q_2 = \max\{Y^r_j, Y^r_{j+1}\}; \tag{3}$$

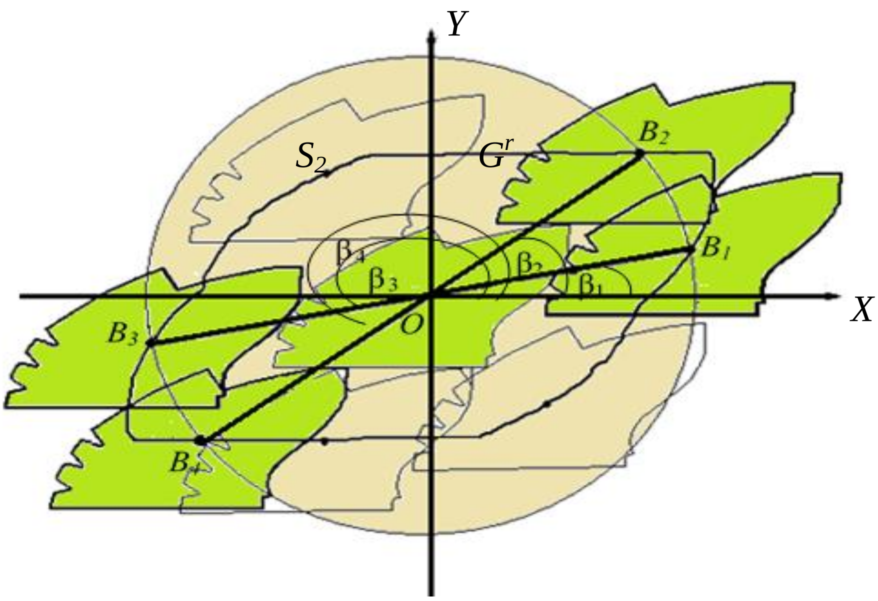

Fig. 3. HVFDP for flat geometric object S_2

- we find the rotation angles β_i; In our case, then we will have where That is, in order to build a grid for the dense arrangement of flat geometric objects S_1 and S_2, it is necessary to turn the flat geometric object S_1 to the angle α and the flat geometric object S_2 to the angle β_i; To find admissible values of the angle of rotation β_i of a flat geometric object S_2 for a given angle of rotation α of a flat geometric object S_1, it is necessary to find the points of intersection of a circle with a radius and with a center at the origin of the coordinates of the HVFDP $G^r\{X^r_j, Y^r_j\}$. And this problem can be easily solved by having an algorithm for finding points intersection of a polygon with a circle of a given radius.

Formulation of the problem. Let us have a polygon G^r with the coordinates of the vertices $G^r_j\{X^r_j, Y^r_j\}$, $j=1,2..n$, and a circle with the center at the origin and the radius R. Find all points of intersection of polygon G^r with a circle of radius R, i.e. all points (Xq_i, Yq_i), $i=1,2,..q$, which simultaneously belong to polygon Gr and a circle with center at the origin and radius R.

The following points can be distinguished in the algorithm for solving this problem:

1. $j = 0$;

2. Select the j-th side of the polygon. The equation of the straight line passing through the j-th side of the polygon G^r with the coordinates of the vertices (Xr_j, Yr_j) and (Xr_{j+1}, Yr_{j+1}) has the form: $A_j x + B_j y + C_j = 0$, де $A_j = Y^r_{j+1} - Y^r_j$; $B_j = X^r_j - X^r_{j+1}$;. $i = 1..m-1$; $C_j = X^r_{j+1} Y^r_j - Y^r_{j+1} X^r_j$;

Let's find the distance D_j from the center of the circle to the straight line

$$A_j x + B_j y + C_j = 0: \; D_j = \frac{|C_j|}{\sqrt{A_j^2 + B_j^2}} \quad (4)$$

A necessary condition for the intersection of the j-th side of the polygon G^r with a circle is that $D_j \leq R$. If this condition is met, then we proceed to point 3, otherwise we return to the beginning of this point;

3. Find the points of intersection of the straight line $A_j x + B_j y + C_j = 0$ with the circle $x^2 + y^2 = R^2$. Consider three cases:

a) a line $A_j x + B_j y + C_j = 0$ parallel to the OX axis. Then $A_j=0$. The equation of the straight line will have the form $B_j y + C_j = 0$. Having solved the system of equations

$$\begin{cases} B_j y + C_j = 0 \\ x^2 + y^2 = R^2 \end{cases}$$

we will find two roots:

$$\begin{array}{ll} X_1 = \sqrt{R^2 - \dfrac{C^2}{B^2}}; & Y_1 = -\dfrac{C}{B}; \\ X_2 = -\sqrt{R^2 - \dfrac{C^2}{B^2}}; & Y_2 = -\dfrac{C}{B}. \end{array} \quad (5)$$

b) a line $A_j x + B_j y + C_j = 0$ parallel to the OY axis. Then $B_j=0$. The equation of the straight line will have the form: $A_j x + C_j = 0$. Having solved the system of equations:

$$\begin{cases} A_j x + C_j = 0 \\ x^2 + y^2 = R^2 \end{cases}$$

we will find two roots:

$$X_1 = \sqrt{R^2 - \frac{C^2}{B^2}}; \quad Y_1 = -\frac{C}{B}; \quad \text{(6)}$$
$$X_2 = -\sqrt{R^2 - \frac{C^2}{B^2}}; \quad Y_2 = -\frac{C}{B}.$$

c) $A_j \neq 0$ and $B_j \neq 0$. Having solved the system of equations: $\begin{cases} A_j x + B_j y + C_j = 0 \\ x^2 + y^2 = R^2 \end{cases}$,

we find two roots:

$$X_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}; \quad Y_1 = -\frac{Ax + C}{B}; \quad \text{(7)}$$
$$X_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}; \quad Y_2 = -\frac{Ax + C}{B},$$

where:

$$a = (1 + \frac{A_j{}^2}{B_j{}^2}); \quad b = \frac{2A_j \cdot C_j}{B}; \quad c = \frac{C_j{}^2}{B_j{}^2} - R^2.$$

We check whether the sought roots satisfy the following conditions:

$T_1 \le X_i \le T;$ $\quad Q_1 \le Y_i \le Q_2.$ (8) where *i=1,2* and T_1, T_2, Q_1, Q_2 are determined by relations (3). If the roots (5-7) satisfy the condition (8), then these are the coordinates of the points of intersection of the j-th side of the polygon with the circle, otherwise we omit these roots.

5. If $j<m-1$, then go to point 2, otherwise end. In this way, we found the coordinates of all points of intersection of the HVFDP with a circle of radius R. And this gives us the opportunity to find all the permissible angles of rotation β_i of the flat geometric object S_2. By determining the angles of rotation and β_i, you can build dense stacks.

The proposed algorithms for finding rotation angles α and β, respectively, for fixed and moving flat geometric objects when constructing dense stacks are implemented in a software product for constructing dense stacks of two flat geometric objects with different configurations of external contours. An example of a constructed dense stack for two different flat geometric objects S_1 and S_2 is presented in Fig. 4.

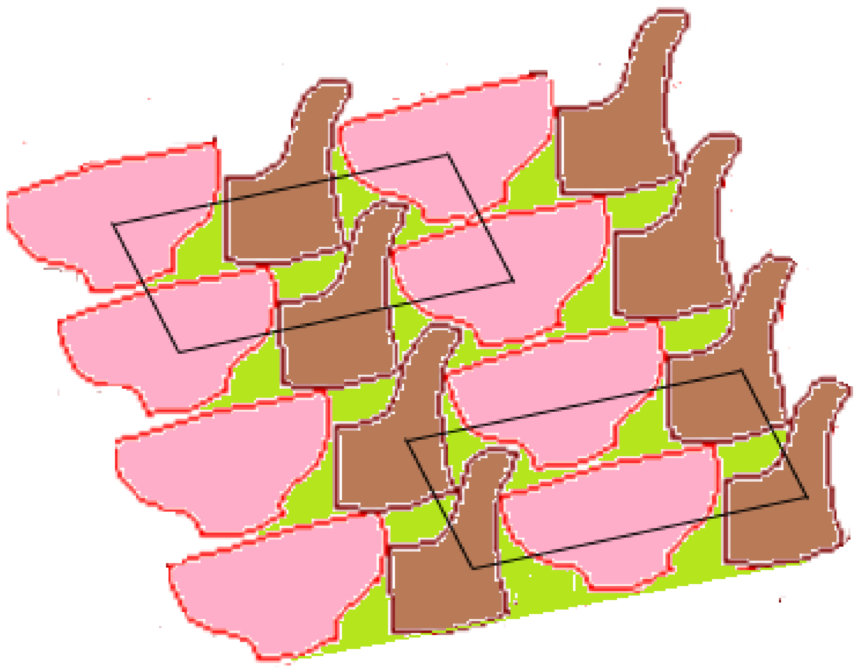

Fig 4. An example of a constructed dense stack for two different flat geometric objects S_1 and S_2

Conclusions. The developed algorithm for finding the rotation angles α and β, respectively, for fixed and moving flat geometric objects when building dense stacks of these objects made it possible to create an effective software product for building dense stacks of two flat geometric objects with different configurations of external contours. The implementation of this software product in the preparatory and cutting production in the shoe industry of light industry will allow to increase the efficiency of the use of materials during cutting and to introduce automated cutting complexes into the cutting production.

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