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DETERMINATION OF THE VELOCITIES OF THE POINTS OF THE THIRD-CLASS MECHANISM WITH THREE LEADING LINKS USING THE GRAPH-ANALYTICAL METHOD

Purpose. To develop sequences of actions and conduct a kinematic study of a complex planar mechanism of the third-class with three leading links by the graph-analytical method to determine the actual values of the angular velocities of individual and linear velocities of all points coinciding with the centres of the kinematic pairs of the mechanism.

Methodology. The kinematic study of the twelve-link mechanism was carried out using the course provisions of the theory of mechanisms and machines about the theory of the structural scheme of mechanical systems and the kinematic analysis of lever mechanisms by the graph-analytical method. Graphic constructions were made in the automated design and drawing system Autocad, which allowed us to bring the accuracy of graphic constructions to the generally accepted level of engineering calculations.

Findings. Using the principles of the theory of mechanisms structure of higher course classes of the theory of mechanisms and machines, a mechanism with three degrees of mobility was considered in the form of three mechanisms, in which the influence of the movement of one leading link with predetermined kinematic parameters on the movement of the mechanism links was successively investigated, for which systems of kinematic equations were compiled with their subsequent solution in the form of graphic constructions. The calculation of the actual angular velocities of those links was obtained, absolute the movement of which is caused by their kinematic attachment to the stationary riser of the mechanism. Calculation of the linear velocities of the points that coincide with the geometric centres of the rotational kinematic pairs of the driven links of the mechanism of the third-class with three leading cranks was performed.

Originality. A plan was developed, and a sequence of actions was implemented, which made it possible to perform kinematic studies and determine the linear velocities of the points that coincide with the geometric centres of the rotational kinematic pairs of links of a complex twelve-link mechanism of the third-class with three leading links using the graph-analytical method. A sequence of studies specially developed for such a planar mechanism with three cranks made it possible to determine the actual values of the angular velocities of the links, the absolute movement of which is determined by their kinematic connection to the stationary body. This made it possible to compile systems of vector kinematic equations to determine the linear velocities of the points of the mechanism of the third-class with three leading links and solve them graphically.

Practical value. The numerical values of the kinematic parameters were obtained and compared with the parameters of the same mechanism, which were calculated using the method of mathematical modelling in the Mathcad software environment. The coincidence of the results of the research performed by two different methods with 95 per cent probability was confirmed, which is a simultaneous confirmation of the credibility of the results obtained by such methods of analysis. Expert research of complex mechanical systems using the graph-analytical method is recommended if engineering calculations are obtained using mathematical modelling technologies.

Keywords: *third-class mechanism, kinematic analysis, graph-analytical method, kinematic study*

Introduction. The basis of a technological machine, according to the definition of the theory of mechanisms and machines, is a mechanism (set of mechanisms), the main function of which is to transform the predetermined movement of the leading link or several links, the movement of which is known, into the necessary movement of the points where the working bodies are located, which ensure the execution of the technological operation. Innovations in technological operations require their provision to create the movement of the working bodies of the machine with geometrically complex trajectories and the necessary laws of motion, taking into account the technological speeds at which they are performed in a short period of time. For successful use in high-speed technological equipment, the mechanisms must have high reliability, durability, and the ability to withstand long and significant loads. These requirements are fully met by mechanisms with rigid links that are kinematically interconnected by lower-class kinematic pairs.

Complex lever mechanisms of the highest class, in contrast to the lower ones, in fact, can theoretically provide any necessary movement of the machine's working organs according to the geometric parameters and laws of motion. In order to ensure specific requirements regarding the laws of movement of the working links of the mechanism, for example, such as the movement with stops of certain links for the time required to perform a technological operation for the cycle of movement of the main shaft, additional driving links are structurally introduced into the mechanisms, which leads to an increase in the degree of mobility of the mechanism to two, three or more units.

The basis of complex high-class lever mechanisms is structural groups of links of the third and higher classes, which are formed by a certain number of links connected by kinematic pairs and structurally (for the specific case of a high-class mechanism) cannot be divided into simpler structural formations according to the theory of the structural schemes of mechanisms of the theory of mechanisms and machines course.

The improvement of the existing and the design of new technological machines based on mechanisms are possible on the condition that their structural [1], kinematic [2], dynamic [3] research has been carried out in advance and taking into account the specifics of those industries where they are used, for example, in the automotive industry [4], mining [5] or in other spheres [6].

Literature review. Kinematic studies of mechanical systems are divided into two main types of problems: analysis and synthesis problems. Analysis problems are characterised by the fact that the research on kinematic parameters of the driven links is carried out according to the previously known parameters of the leading links of the mechanism, such tasks are also called "direct kinematics" [7]. Research in the "reverse" direction requires establishing the relationship between the kinematic parameters of the leading and driven links in the form of equations and their systems and solving them under the conditions of given technological and functional parameters, which, of course, prompts the solution of synthesis problems, for example, in the early stages of carrying out the structural arrangement of the mechanism [8] or after the kinematic calculations have been performed [9].

Perfecting and modernisation of existing technological equipment are carried out on the basis of solving analysis problems. In the work [10], an eight-link hinged mechanism was investigated, the design of the machine for processing parts was analysed in the SolidWorks automated design system, and the result of increasing the reliability of the machine during operation was obtained while increasing its productivity.

In the work [11], using Mathcad software, the authors performed a kinematic analysis of a complex planar mechanism used in technological equipment, in which a stoppage of the working body is observed in the time period according to the requirements. Based on the results of the analysis, the reason for the incomplete technological stop of the working link was established, and recommendations were given to eliminate the identified defect in the mechanism's design.

The kinematic analysis of a complex planar eight-link mechanism is the subject of work [12], in which the authors present a mathematical model that allows dividing one velocity matrix and one acceleration matrix into two matrices of different orders, which simplifies the process of their solving.

In the work [13], the tasks of analysing a number of complex multi-link planar mechanisms were performed using SolidWorks software. The analysis of seven-link spatial mechanisms without redundant links made it possible to design a machine drive configuration in a machine for machining parts that ensures the rotation of the drive shaft with its simultaneous reciprocating motion [14].

In the work [15], the kinematic synthesis of a six-link hinged mechanism with one translational kinematic pair was carried out, the mechanism for shaping the cross-section of the conveyor belt of the minimum possible overall dimensions with the smallest possible driving force was designed, and recommendations for their implementation were given.

The simultaneous problem-solving of analysis and synthesis of a complex planar articulated lever mechanism of the fourth class as the basis of a double-jaw crushing machine was presented in the work [16]. According to the required technological parameters, the authors of the paper developed a design of the machine mechanism with complex jaw movement using Autocad computer-aided design and drawing systems, analytical calculations of the fourth-class flat lever mechanism and determination of its rational geometric parameters were performed using mathematical modelling in the Mathcad software environment.

Particular attention is paid to mechanisms with several leading links in the articles. Mechanisms with two leading links or parallel hinged chain mechanisms [17] are used in works known as parallel works for precise positioning and alignment. In this work, the kinematic analysis of such mechanisms is performed using geometric and analytical methods, the results of which are rechecked to confirm the obtained results. The advantage of the method is that it can be extended to mechanisms with a larger number of leading links. Another work [18] presents a kinematic analysis of a fourth-class mechanism, which was performed by mathematical modelling of the kinematic scheme and mechanism parameters in the MathCad program and by the graph-analytical method to verify the obtained results. The kinematic analysis of mechanisms with two leading links for a parking robot was carried out in the work [19]. On the basis of multiple kinematic performance indicators, the search for the optimal combination of structural parameters of the robot mechanism in the permissible area was performed in order to solve the problem of optimising energy saving.

The works [20, 21] consider the problems of kinematic analysis of mechanisms with three leading and other driven links.

Unsolved aspects of the problem. According to the theory of the structural schemes of planar mechanisms of the course of the theory of mechanisms and machines, complex mechanisms of the highest class with one leading link structurally consist of one initial mechanism (a set of a leading link and a fixed link connected by one kinematic pair) and structural formations of links whose class is three or more. Accordingly, mechanisms of a higher class with two, three or more leading links in their structure have two, three or more initial mechanisms. The class of the mechanism is determined by the maximum class of structural groups of links formed by the driven links of the mechanism. For mechanisms with a structural group of links of the third class and one complex link (base link), there are developed methods of kinematic research that allow them to be carried out in a graph-analytical manner. A different situation is experienced when performing similar studies for mechanisms with two or more leading links of the third class with two or more complex links in the structure or of the fourth and higher class if such research is actually possible for certain mechanisms: in each case of such mechanical systems, it is necessary to develop a plan for their investigation in a graphical manner so as to take into account their individual structural features.

The relevance of conducting kinematic research of mechanisms of the third and higher classes in a graph-analytical way using Autocad computer-aided design and drawing systems is confirmed by the accuracy of the results obtained in this way, which can be compared with the accuracy of the results of similar studies using mathematical modelling, for example, in the Mathcad software environment, and therefore the results of graph-analytical studies can be used to perform verification calculations.

The purpose of the article. To develop a research plan and conduct a kinematic analysis of the third-class planar mechanism with three leading links using the graph-analytical method by means of the Autocad computer-aided design and drawing system to determine the actual values of the angular velocities of individual links of the mechanism and the linear velocities of all points coinciding with the centres of the kinematic pairs of the mechanism. To obtain numerical values of kinematic parameters and perform their comparative analysis with similar parameters of the same mechanism, which were calculated using the method of mathematical modelling in the Mathcad software environment.

Methods. The article presents a kinematic analysis of a complex planar mechanism of the third class with three leading links, which was performed using the provisions of the course on the theory of mechanisms and machines on the theory of the structural scheme of mechanical systems and the kinematic analysis of lever mechanisms by the graph-analytical method. The mechanism with three degrees of mobility was presented in the form of three mechanisms, in which the influence of the movement of one leading link with predetermined kinematic parameters on the movement of other driven links of the mechanism was successively investigated, which allowed compiling systems of vector equations with their subsequent solution in the form of graphic constructions. The calculation of the actual angular velocities of those links whose absolute motion is due to their kinematic connection to a fixed riser of the mechanism was obtained, and the linear velocities of the points coinciding with the geometric centres of the rotational kinematic pairs of driven links of the third-class mechanism with three driving cranks were calculated. The graphical constructions were made using a computer-aided design and drawing Autocad, which allowed the accuracy of graphical constructions and research results to be brought to the generally accepted level of engineering calculations.

Presentation of the main material and scientific results. The planar mechanism of the third class (Fig. 1) [11] consists of three leading cranks from *1* to *3*, which on the one hand, are connected to the fixed body of the mechanism (link *0*), and on the other hand, are connected to the structural group of links of the third class of the fourth order (links *4* to *9*) and other driven links *10* and *11*, which form the structural group of links of the second class. In the mechanism, links from *4* to *8* and *10* have planar parallel motion, links from *1* to *3*, *9* and *11* have rotational motion. Mechanisms of the first class (sets of links *0*, *1*; *0*, *2* and *0*, *3*) together with structural groups of links of the third and second classes and fifteen rotary kinematic pairs of the fifth class form a mechanism of the third class with three degrees of motion relative to the fixed body. The presence of three leading links in the structure of the mechanism causes that in the group of links directly attached to them there are five links with complex planar motion. Under the condition of the study, the kinematic parameters of three points from A_1 to A_3 coinciding with the centres of the kinematic pairs by which the leading cranks are connected to the three driven connecting rods of the structural group of links of the third class are given. The task of studying such a mechanism is complicated by the fact that three connecting rods from *4* to *6* are connected by kinematic pairs to the other two connecting rods *7*, *8*, which are made in the form of complex links, kinematically connected to each other and have no connection to the fixed body of the mechanism. For the points that coincide with the centres of the kinematic pairs of the structural group of links of the third class, it is not possible to draw up kinematic equations and solve them graphically. The kinematic analysis of a complex planar mechanism by the graph-analytical method is based on the fact that the determination of the kinematic parameter of a point coinciding with the centre of a kinematic pair formed by two connecting rods should take into account the fact that the parameters of the other two points belonging to two different connecting rods are known, which is not observed in our case. So, for example, for point *B*, which coincides with the kinematic pair that is formed by connecting rods *4* and *7*, in the system of kinematic vector equations, the kinematic parameters of points

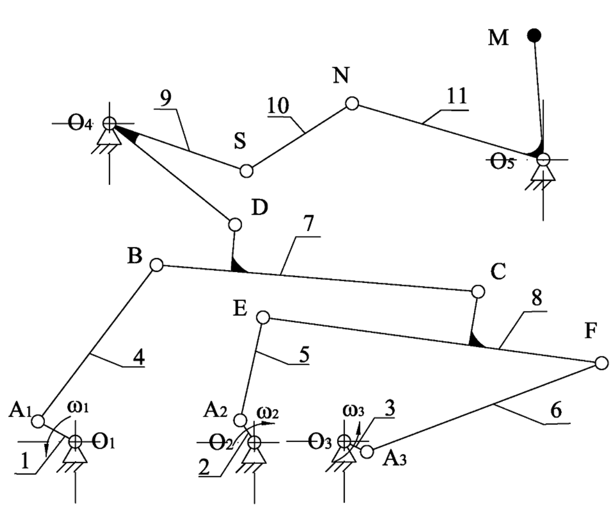

Fig. 1. Kinematic diagram of the third-class mechanism

A_1, C or D are used. If the kinematic parameters of point A_1 are calculated on the condition that it belongs to the leading crank *1*, then it is impossible to calculate the parameters of point C or D – the kinematic parameters of links *8* and *9* are unknown. A similar situation is observed when trying to draw up and solve the vector equations for points E and F. It is also impossible to determine the position of the "special points" of the Assur group [1], which are used in the method of kinematic research, which is given in the literature for the investigation of third-class mechanisms, the presence of six links from *4* to *9* in the structural group of the third class, two of which are complex (links *7*, *8*) does not allow one to do it.

For the kinematic analysis of a third-class mechanism with three leading links, we develop a research sequence that is individually characteristic of its structure. If crank *1* is chosen as the leading link with the leading links *2* and *3* conditionally stopped, we will have a kinematic diagram of the mechanism in which points A_2 and A_3 are fixed, which will allow us to find out how the angular velocity ω_1 of crank *1* affects the values of the absolute angular velocities ω_9 and ω_{11} of the rocker arms *9* and *11* of the mechanism. We repeat the same for the case when crank *2* is the leading link (if $\omega_1 = \omega_3 = 0$, s^{-1}) and for link *3* (if $\omega_1 = \omega_2 = 0$, s^{-1}) and determine the angular velocities of links *9* and *11*. Algebraically, we add the values of the respective angular velocities ω_9, ω_{11} of links *9*, *11* and obtain the actual angular velocities of these links for the mechanism under research. The real values of the angular velocities of links *9* and *11* determined in this way will allow us to perform a kinematic analysis of the third-class mechanism with three leading links in a graph-analytical way.

The study started in the case when crank *1* was the only leading link in the mechanism. The set parameters for the kinematic analysis of such third-class mechanism are the angular velocity of leading link *1* (ω_1 = const, s^{-1}, the other leading links *2*, *3* are conditionally fixed: $\omega_2 = \omega_3 = 0$, s^{-1}) and the scale of lengths (Kl, m/mm) of the kinematic scheme of the mechanism (Fig. 1). Provided that there is one leading crank 1, the mechanism remains structurally a third-class mechanism with a degree of mobility of one relative to the fixed body.

Using the property of complex planar mechanisms with one leading link to change class, under the condition that another link is selected as a possible moving link of the initial mechanism [18]. We conventionally select link *6* as the moving link of the initial mechanism. Taking into account the fact that links *2* and *3* are conditionally fixed, the mechanism structurally takes the form of a second-class mechanism with a sequentially parallel connection of four structural groups of second-class links, which include the following links in pairs: *5*, *8*; *7*, *9*; *1*, *4* and *10*, *11*.

We choose an optional scale and begin the graphical construction of the velocity plan (Fig. 2), setting the direction of rotation of link *6*, for example, against the clockwise direction.

Vector $\overrightarrow{Pf}$ on the velocity plan is chosen to be of arbitrary length and is plotted in the perpendicular direction to the segment A_3F. Further construction of the velocity plan is performed in the following sequence:

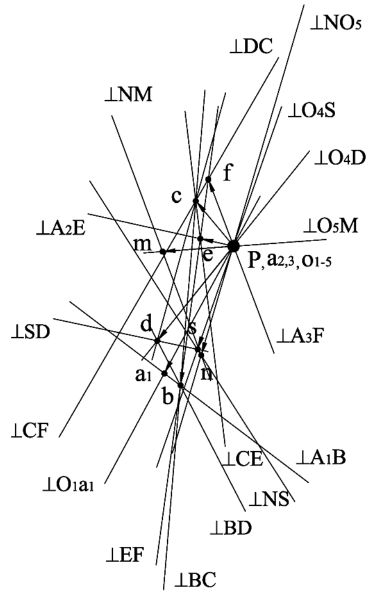

Fig. 2. Velocity plan for a mechanism with a leading crank 1 (ω_1 = const, $\omega_2 = \omega_3 = 0$, s^{-1})

1. We build the velocity vector of point E ($\overrightarrow{Pe}$), the position of point "e" is found on the velocity plan from the graphical solution of the system of vector equations

$$\begin{cases} \vec{V}_E = \vec{V}_F + \vec{V}_{E;F} \\ \vec{V}_E = \vec{V}_{A_2} + \vec{V}_{E;A_2} \end{cases},$$

where $\vec{V}_F$, $\vec{V}_{A_2}$ are vectors of absolute velocities points F and A_2; $\vec{V}_{E;F} \perp E;F$, $\vec{V}_{E;A_2} \perp E;A_2$ are the corresponding vectors of relative velocities.

2. Velocity vector $\vec{V}_C$ of point C ($\overrightarrow{Pc}$) on the plan is determined by solving a system of vector equations

$$\begin{cases} \vec{V}_C = \vec{V}_F + \vec{V}_{C;F} \\ \vec{V}_C = \vec{V}_E + \vec{V}_{C;E} \end{cases},$$

where $\vec{V}_F$, $\vec{V}_E$ are vectors of absolute velocities points F and E; $\vec{V}_{C;F} \perp C;F$, $\vec{V}_{C;E} \perp C;E$ are the corresponding vectors of relative velocities.

3. We build the velocity vector of point D ($\overrightarrow{Pd}$), the position of point "d" is found on the velocity plan from the graphical solution of the system of vector equations

$$\begin{cases} \vec{V}_D = \vec{V}_C + \vec{V}_{D;C} \\ \vec{V}_D = \vec{V}_{O_4} + \vec{V}_{D;O_4} \end{cases},$$

where $\vec{V}_C$, $\vec{V}_{O_4}$ are vectors of absolute velocities points C and O_4; $\vec{V}_{D;C} \perp D;C$, $\vec{V}_{D;O_4} \perp D;O_4$ are the corresponding vectors of relative velocities.

4. Velocity vector $\vec{V}_B$ of point B ($\overrightarrow{Pb}$) on the plan is determined by solving a system of vector equations

$$\begin{cases} \vec{V}_B = \vec{V}_D + \vec{V}_{B;D} \\ \vec{V}_B = \vec{V}_C + \vec{V}_{B;C} \end{cases},$$

where $\vec{V}_D$, $\vec{V}_C$ are vectors of absolute velocities points D and C; $\vec{V}_{B;D} \perp B;D$, $\vec{V}_{B;C} \perp B;C$ are the corresponding vectors of relative velocities.

5. We build the velocity vector of point S $(\overrightarrow{Ps})$ by solving a system of vector equations

$$\begin{cases} \vec{V}_S = \vec{V}_D + \vec{V}_{S;D} \\ \vec{V}_S = \vec{V}_{O_4} + \vec{V}_{S;O_4} \end{cases},$$

where $\vec{V}_D$, $\vec{V}_{O_4}$ are vectors of absolute velocities points D and O_4; $\vec{V}_{S;D} \perp S;D$, $\vec{V}_{S;O_4} \perp S;O_4$ are the corresponding vectors of relative velocities.

6. Velocity vector $\vec{V}_N$ of point N $(\overrightarrow{Pn})$ on the plan is determined by solving a system of vector equations

$$\begin{cases} \vec{V}_N = \vec{V}_S + \vec{V}_{N;S} \\ \vec{V}_N = \vec{V}_{O_5} + \vec{V}_{N;O_5} \end{cases},$$

where $\vec{V}_S$, $\vec{V}_{O_5}$ are vectors of absolute velocities points S and O_5; $\vec{V}_{N;S} \perp N;S$, $\vec{V}_{N;O_5} \perp N;O_5$ are the corresponding vectors of relative velocities.

7. We build the velocity vector of point M $(\overrightarrow{Pm})$ by solving a system of vector equations

$$\begin{cases} \vec{V}_M = \vec{V}_N + \vec{V}_{M;N} \\ \vec{V}_M = \vec{V}_{O_5} + \vec{V}_{M;O_5} \end{cases},$$

where $\vec{V}_N$, $\vec{V}_{O_5}$ are vectors of absolute velocities points N and O_5; $\vec{V}_{M;N} \perp M;N$, $\vec{V}_{M;O_5} \perp M;O_5$ are the corresponding vectors of relative velocities.

8. We build the velocity vector of point A_1 $(\overrightarrow{Pa_1})$, the position of point "a_1" can be found on the velocity plan from the graphical solution of the system of vector equations

$$\begin{cases} \vec{V}_{A_1} = \vec{V}_B + \vec{V}_{A_1;B} \\ \vec{V}_{A_1} = \vec{V}_{O_1} + \vec{V}_{A_1;O_1} \end{cases},$$

where $\vec{V}_B$, $\vec{V}_{O_1}$ are vectors of absolute velocities points B and O_1; $\vec{V}_{A_1;B} \perp A_1;B$, $\vec{V}_{A_1;O_1} \perp A_1;O_1$ are the corresponding vectors of relative velocities.

9. We determine the velocity of point A_1, m/s

$$V_{A_1} = \omega_1 \cdot l_{O_1A_1} = 100 \cdot 0.012 = 1.2,$$

where the angular velocity of the crank *1* $\omega_1 = +100$, s^{-1}; $l_{O_1A_1}$ – link length O_1A_1, m.

10. Scale of the speed plan, $\frac{\text{m/s}}{\text{mm}}$ (for the case when $\omega_1 = 100 = \text{const}$, $\omega_2 = 0$, $\omega_3 = 0$, s^{-1})

$$K_V = \frac{V_{A_1}}{P_{a_1}} = \frac{1.2}{15.46} = 0.078,$$

where $P_{a_1} = 15.46$ mm – the length of the part on the velocity plan.

11. We calculate the angular velocities of the links *9* and *11*, s^{-1}

$$\omega_9 = \frac{V_D}{l_{O_4D}} = \frac{Pd \cdot K_V}{l_{O_4D}} = \frac{13.0 \cdot 0.078}{0.045} = -22.50;$$

$$\omega_{11} = \frac{V_{N;O_5}}{l_{NO_5}} = \frac{Pn \cdot K_V}{l_{NO_5}} = \frac{12.12 \cdot 0.078}{0.055} = 17.19,$$

where l_{O_4D}, l_{NO_5} are link lengths O_4D, NO_5, m; Pd, Pn – the length of the parts on the velocity plan, mm.

The "–" sign for the values of the angular velocities of the links indicates that the direction of the angular velocity of the link coincides with the direction of clockwise movement, the "+" sign indicates that the direction of the angular velocity of the link is against the clockwise direction.

We investigate the effect of the movement of crank *2* on the kinematic parameters of the driven links of the mechanism. Links *1*, *3* are considered to be conditionally fixed ($\omega_2 = \text{const}$, s^{-1}, $\omega_1 = \omega_3 = 0$, s^{-1}). We conditionally choose link 4 as the moving link of the initial mechanism. Taking into account that links *1* and *3* are conditionally fixed, the mechanism structurally takes the form of a second-class mechanism with a sequentially parallel connection of four structural groups of second-class links, which include the following links in pairs: *7*, *9*; *10*, *11* and *6*, *8*; *2*, *5*.

On the velocity plan (Fig. 3), the vector $\overrightarrow{Pb}$ we choose an arbitrary length and place it perpendicular to the segment A_1. The direction of rotation of link *4* is set arbitrarily, for example, in the clockwise direction.

We continue to build the velocity plan in the following sequence:

1. We build the velocity vector of point D $(\overrightarrow{Pd})$, the position of point "d" can be found on the velocity plan from the graphical solution of the system of vector equations

$$\begin{cases} \vec{V}_D = \vec{V}_B + \vec{V}_{D;B} \\ \vec{V}_D = \vec{V}_{O_4} + \vec{V}_{D;O_4} \end{cases}, \tag{1}$$

where $\vec{V}_B$, , $\vec{V}_{O_4}$ are vectors of absolute velocities points B and O_4; $\vec{V}_{D;B} \perp D;B$ and, $\vec{V}_{D;O_4} \perp D;O_4$ are the corresponding vectors of relative velocities.

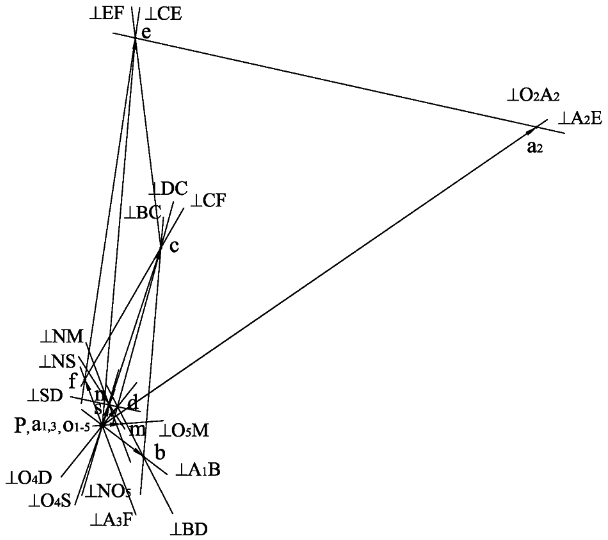

Fig. 3. Velocity plan for a mechanism with a leading crank 2 ($\omega_2 = \text{const}$, $\omega_1 = \omega_3 = 0$, s^{-1})

2. Velocity vector $\vec{V}_C$ of point C $(\overrightarrow{Pc})$ on the plan is determined by solving a system of vector equations

$$\begin{cases} \vec{V}_C = \vec{V}_B + \vec{V}_{C;B} \\ \vec{V}_C = \vec{V}_D + \vec{V}_{C;D} \end{cases}, \tag{2}$$

where $\vec{V}_B$, $\vec{V}_D$ are vectors of absolute velocities points B and D; $\vec{V}_{C;B} \perp C;B$, $\vec{V}_{C;D} \perp C;D$ are the corresponding vectors of relative velocities.

3. We build the velocity vector of point S $(\overrightarrow{Ps})$, the position of point "s" can be found on the velocity plan from the graphical solution of the system of vector equations

$$\begin{cases} \vec{V}_S = \vec{V}_D + \vec{V}_{S;D} \\ \vec{V}_S = \vec{V}_{O_4} + \vec{V}_{S;O_4} \end{cases}, \tag{3}$$

where $\vec{V}_D$, $\vec{V}_{O_4}$ are vectors of absolute velocities points D and O_4; $\vec{V}_{S;D} \perp S;D$, $\vec{V}_{S;O_4} \perp S;O_4$ are the corresponding vectors of relative velocities.

4. Velocity vector $\vec{V}_N$ of point "N" $(\overrightarrow{Pn})$ on the plan is determined by solving a system of vector equations

$$\begin{cases} \vec{V}_N = \vec{V}_S + \vec{V}_{N;S} \\ \vec{V}_N = \vec{V}_{O_5} + \vec{V}_{N;O_5} \end{cases}, \tag{4}$$

where $\vec{V}_S$, $\vec{V}_{O_5}$ are vectors of absolute velocities points S and O_5; $\vec{V}_{N;S} \perp N;S$, $\vec{V}_{N;O_5} \perp N;O_5$ are the corresponding vectors of relative velocities.

5. We build the velocity vector of point M $(\overrightarrow{Pm})$ by solving a system of vector equations

$$\begin{cases} \vec{V}_M = \vec{V}_N + \vec{V}_{M;N} \\ \vec{V}_M = \vec{V}_{O_5} + \vec{V}_{M;O_5} \end{cases}, \tag{5}$$

where $\vec{V}_N$, $\vec{V}_{O_5}$ are vectors of absolute velocities points N and O_5; $\vec{V}_{M;N} \perp M;N$, $\vec{V}_{M;O_5} \perp M;O_5$ are the corresponding vectors of relative velocities.

6. Velocity vector $\vec{V}_F$ of point F $(\overrightarrow{Pf})$ on the plan is determined by solving a system of vector equations

$$\begin{cases} \vec{V}_F = \vec{V}_C + \vec{V}_{F;C} \\ \vec{V}_F = \vec{V}_{A_3} + \vec{V}_{F;A_3} \end{cases},$$

where $\vec{V}_C$, $\vec{V}_{A_3}$ are vectors of absolute velocities points C and A_3; $\vec{V}_{F;C} \perp F;C$, $\vec{V}_{F;A_3} \perp F;A_3$ are the corresponding vectors of relative velocities.

7. We build the velocity vector of point E $(\overrightarrow{Pe})$ by solving a system of vector equations

$$\begin{cases} \vec{V}_E = \vec{V}_F + \vec{V}_{E;F} \\ \vec{V}_E = \vec{V}_C + \vec{V}_{E;C} \end{cases},$$

where $\vec{V}_F$, $\vec{V}_C$ are vectors of absolute velocities points F and C; $\vec{V}_{E;F} \perp E;F$, $\vec{V}_{E;C} \perp E;C$ are the corresponding vectors of relative velocities.

8. Velocity vector $\vec{V}_{A_2}$ of point A_2 $(\overrightarrow{Pa_2})$ on the plan is determined by solving a system of vector equations

$$\begin{cases} \vec{V}_{A_2} = \vec{V}_E + \vec{V}_{A_2;E} \\ \vec{V}_{A_2} = \vec{V}_{O_2} + \vec{V}_{A_2;O_2} \end{cases},$$

where $\vec{V}_E$, $\vec{V}_{O_2}$ are vectors of absolute velocities points E and O_2; $\vec{V}_{A_2;E} \perp A_2;E$, $\vec{V}_{A_2;O_2} \perp A_2;O_2$ are the corresponding vectors of relative velocities.

9. We determine the velocity of point A_2, m/s

$$V_{A_2} = \omega_2 \cdot l_{O_2A_2} = 200 \cdot 0.007 = 1.4,$$

where the angular velocity of the crank *2* $\omega_2 = -200$, s^{-1}; $l_{O_2A_2}$ – link length O_2A_2, m.

10. Scale of the speed plan, $\frac{\text{m/s}}{\text{mm}}$ (for the case when $\omega_2 = 200 = \text{const}$, $\omega_1 = 0$, $\omega_3 = 0$, s^{-1})

$$K_V = \frac{V_{A_2}}{P_{a_2}} = \frac{1.4}{125.64} = 0.011,$$

where $P_{a_2} = 125.64$, mm is the length of the part on the velocity plan.

11. We calculate the angular velocities of the links *9* and *11*, s^{-1}

$$\omega_9 = \frac{V_D}{l_{O_4D}} = \frac{Pd \cdot K_V}{l_{O_4D}} = \frac{5.80 \cdot 0.011}{0.045} = 1.42;$$

$$\omega_{11} = \frac{V_{N;O_5}}{l_{NO_5}} = \frac{Pn \cdot K_V}{l_{NO_5}} = \frac{5.3 \cdot 0.011}{0.055} = -1.06,$$

where l_{O_4D}, l_{NO_5} are lengths of links O_4D, NO_5, m; Pd, Pn – lengths of parts on the velocity plan, mm.

We investigate the effect of the movement of crank *3* on the kinematic parameters of the driven links of the mechanism. Links *1*, *2* are considered to be conditionally fixed ($\omega_3 = \text{const}$, $\omega_1 = \omega_2 = 0$, s^{-1}). We conditionally choose link *4* as the moving link of the initial mechanism. Taking into account that links *1* and *2* are conditionally fixed, the mechanism structurally takes the form of a second-class mechanism with a sequentially parallel connection of four structural groups of second-class links, which include the following links in pairs: *7*, *9*; *10*, *11* and *5*, *8*; *3*, *6*.

On the velocity plan (Fig. 4), the vector $\overrightarrow{Pb}$, we choose an arbitrary length and place it perpendicular to the segment A_1B. The direction of rotation of link *4* is set arbitrarily, for example, in the clockwise direction. It should be noted that in this case, the sequence of the research coincides with the sequence of constructing the velocity plan for the case when the influence of the movement of the second crank on the parameters of the movement of the driven links of the mechanism was studied, so the construction of the plan we perform according to the systems of vector equations from (1) to (5) and on the research items from 1 to 5 (for $\omega_2 = \text{const}$, $\omega_1 = \omega_3 = 0$, s^{-1}).

We continue to build the velocity plan in the following sequence:

6. Velocity vector $\vec{V}_E$ of point E $(\overrightarrow{Pe})$ on the plan is determined by solving a system of vector equations

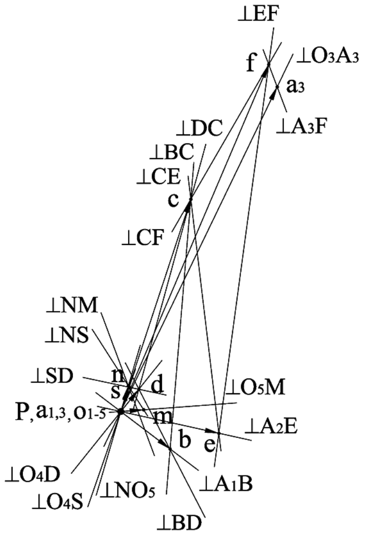

Fig. 4. Velocity plan for a mechanism with a leading crank 3 ($\omega_3 = \text{const}$, $\omega_1 = \omega_2 = 0$, s^{-1})

$$\begin{cases} \vec{V}_E = \vec{V}_{A_2} + \vec{V}_{E;A_2} \\ \vec{V}_E = \vec{V}_C + \vec{V}_{E;C} \end{cases},$$

where $\vec{V}_{A_2}$, $\vec{V}_C$ are vectors of absolute velocities points A_2 and C; $\vec{V}_{E;A_2} \perp E;A_2$, $\vec{V}_{E;C} \perp E;C$ are the corresponding vectors of relative velocities.

7. We build the velocity vector of point F ($\overrightarrow{Pf}$) by solving a system of vector equations

$$\begin{cases} \vec{V}_F = \vec{V}_C + \vec{V}_{F;C} \\ \vec{V}_F = \vec{V}_E + \vec{V}_{F;E} \end{cases},$$

where $\vec{V}_C$, $\vec{V}_E$ are vectors of absolute velocities points C and E; $\vec{V}_{F;C} \perp F;C$, $\vec{V}_{F;E} \perp F;E$ are the corresponding vectors of relative velocities.

8. Velocity vector $\vec{V}_{A_3}$ of point A_3 ($\overrightarrow{Pa_3}$) on the plan is determined by solving a system of vector equations

$$\begin{cases} \vec{V}_{A_3} = \vec{V}_F + \vec{V}_{A_3;F} \\ \vec{V}_{A_3} = \vec{V}_{O_3} + \vec{V}_{A_3;O_3} \end{cases},$$

where $\vec{V}_F$, $\vec{V}_{O_3}$ are vectors of absolute velocities points F and O_3; $\vec{V}_{A_3;F} \perp A_3;F$, $\vec{V}_{A_3;O_3} \perp A_3;O_3$ are the corresponding vectors of relative velocities.

9. We determine the velocity of point A_3, m/s

$$V_{A_3} = \omega_3 \cdot l_{O_3A_3} = 300 \cdot 0.007 = 2.1,$$

where the angular velocity of the crank *3* $\omega_3 = +300$, s^{-1}; $l_{O_3A_3}$ – link length O_3A_3, m.

10. Scale of the speed plan, $\frac{\text{m/s}}{\text{mm}}$ (for the case when $\omega_3 = 300 = \text{const}$, $\omega_1 = 0$, $\omega_2 = 0$, s^{-1})

$$K_V = \frac{V_{A_3}}{P_{a_3}} = \frac{2.1}{71.84} = 0.029, \quad (6)$$

where $P_{a_3} = 71.84$ mm is the length of the part on the velocity plan.

11. We calculate the angular velocities of the links *9* and *11*, s^{-1}

$$\omega_9 = \frac{V_{D;O_4}}{l_{O_4D}} = \frac{Pd \cdot K_V}{l_{O_4D}} = \frac{5.73 \cdot 0.029}{0.045} = 3.69;$$

$$\omega_{11} = \frac{V_{N;O_5}}{l_{NO_5}} = \frac{Pn \cdot K_V}{l_{NO_5}} = \frac{5.3 \cdot 0.029}{0.055} = -2.80,$$

where l_{O_4D}, l_{NO_5} are lengths of links O_4D, NO_5, m; Pd, Pn – lengths of parts on the velocity plan, mm.

The results of calculations of the angular velocities of links *9* and *11* are presented in Table 1.

We compare the results of the research with the calculations performed for this mechanism with the same initial kinematic parameters using mathematical modelling in the Mathcad software environment [11]. The error of the calculated angular velocities $\Delta\omega_9$, $\Delta\omega_{11}$ is less than 5 %, which simultaneously confirms the correctness of the results of the study obtained using two different methods:

$$\Delta\omega_9 = \frac{\left|\omega_9 - \omega_9^{an}\right|}{\omega_9^{an}} \cdot 100\,\% = \frac{17.39 - 16.90}{16.90} \cdot 100\,\% = 2.9\,\%;$$

$$\Delta\omega_{11} = \frac{\left|\omega_{11} - \omega_{11}^{an}\right|}{\omega_{11}^{an}} \cdot 100\,\% = \frac{13.33 - 12.80}{12.80} \cdot 100\,\% = 4.1\,\%,$$

where $\omega_9^{an} = -16.90$, s^{-1}; $\omega_{11}^{an} = 12.80$, s^{-1} are the results of the angular velocities of links *9*, *11*, which were obtained using the analytical method of software modelling.

We conclude the study by constructing a velocity plan of a third-class mechanism with three simultaneously moving leading links *1–3* (Fig. 5) on a predetermined scale for the given parameters of the movement of the three leading links and the found values of the absolute instantaneous angular velocities of rocker arms *9* and *11*. We calculate the absolute values of the linear velocities of the points belonging to the links whose instantaneous absolute motion is determined.

The results of the calculations are presented in Table 2.

We arbitrarily choose the scale value of the velocity plan (6) and construct the vectors of absolute velocities of points A_1, A_2, A_3, D, S, N, M, respectively, by parts Pa_1, Pa_2, Pa_3, Pd, Ps, Pn, Pm.

Table 1

Angular velocities of links *9* and *11*

parameters / value	$\omega_1 = +100 = \text{const}$, $\omega_2 = \omega_3 = 0$, s^{-1}	$\omega_2 = -200 = \text{const}$, $\omega_1 = \omega_3 = 0$, s^{-1}	$\omega_3 = +300 = \text{const}$, $\omega_1 = \omega_2 = 0$, s^{-1}	$\omega_1 = +100$, $\omega_2 = -200$, $\omega_3 = +300$, s^{-1}
ω_9, s^{-1}	–22.50	1.42	3.69	–17.39
ω_{11}, s^{-1}	17.19	–1.06	–2.80	13.33

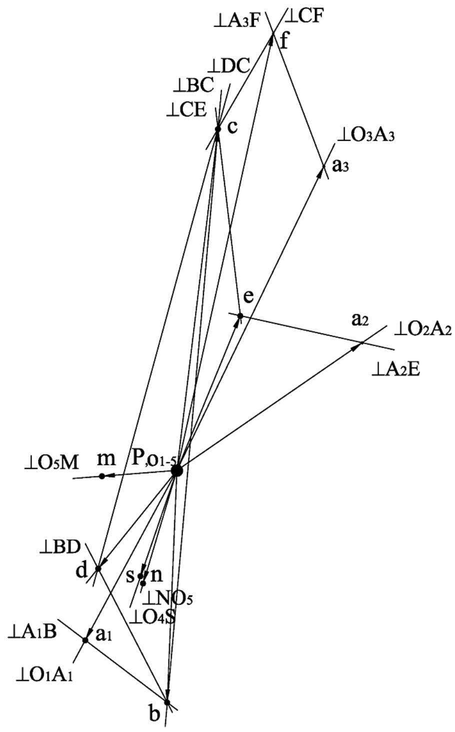

Fig. 5. Velocity plan of a third-class mechanism with three leading links

We continue to build the velocity plan in the following sequence:

1. We build the velocity vector of point B $(\overrightarrow{Pb})$, the position of point "d" can be found on the velocity plan from the graphical solution of the system of vector equations

$$\begin{cases} \vec{V}_B = \vec{V}_{A_1} + \vec{V}_{B;A_1} \\ \vec{V}_B = \vec{V}_D + \vec{V}_{B;D} \end{cases},$$

where $\vec{V}_{A_1}$, $\vec{V}_D$ are vectors of absolute velocities points A_1 and D; $\vec{V}_{B;A_1} \perp A_1B$, $\vec{V}_{B;D} \perp B;D$ are the corresponding vectors of relative velocities.

2. Velocity vector $\vec{V}_C$ of point C $(\overrightarrow{Pc})$ on the plan is determined by solving a system of vector equations

$$\begin{cases} \vec{V}_C = \vec{V}_D + \vec{V}_{C;D} \\ \vec{V}_C = \vec{V}_B + \vec{V}_{C;B} \end{cases},$$

where $\vec{V}_D$, $\vec{V}_B$ are vectors of absolute velocities points D and B; $\vec{V}_{C;D} \perp C;D$, $\vec{V}_{C;B} \perp C;B$ are the corresponding vectors of relative velocities.

3. We build the velocity vector of point E $(\overrightarrow{Pe})$, the position of point "e" can be found on the velocity plan from the graphical solution of the system of vector equations

$$\begin{cases} \vec{V}_E = \vec{V}_{A_2} + \vec{V}_{E;A_2} \\ \vec{V}_E = \vec{V}_C + \vec{V}_{E;C} \end{cases},$$

where $\vec{V}_{A_2}$, $\vec{V}_C$ are vectors of absolute velocities points A_2 and C; $\vec{V}_{E;A_2} \perp E;A_2$, $\vec{V}_{E;C} \perp E;C$ are the corresponding vectors of relative velocities.

4. Velocity vector $\vec{V}_F$ of point F $(\overrightarrow{Pf})$ on the plan is determined by solving a system of vector equations

$$\begin{cases} \vec{V}_F = \vec{V}_{A_3} + \vec{V}_{F;A_3} \\ \vec{V}_F = \vec{V}_C + \vec{V}_{F;C} \end{cases},$$

where $\vec{V}_{A_3}$, $\vec{V}_C$ are vectors of absolute velocities points A_3 and C; $\vec{V}_{F;A_3} \perp F;A_3$, $\vec{V}_{F;C} \perp F;C$ are the corresponding vectors of relative velocities.

The results of calculations of the linear velocities of the points of the third-class mechanism with three lead-

Table 2

Linear velocities of points of the third-class mechanism

Designation of angular velocity	ω_1	ω_2	ω_3	ω_9	ω_9	ω_{11}	ω_{11}
Angular velocity values, s^{-1}	100.00	200.00	300.00	17.39	17.39	13.33	13.33
Designation of link lengths	$l_{O_1A_1}$	$l_{O_2-A_2}$	$l_{O_3A_3}$	l_{O_4-D}	l_{O_4-S}	l_{O_5-N}	l_{O_5-M}
Length values, m	0.012	0.007	0.007	0.045	0.040	0.055	0.035
Linear velocity of a point	$V_{A_1} = \omega_1 \cdot l_{O_1A_1}$	$V_{A_2} = \omega_2 \cdot l_{O_2A_2}$	$V_{A_3} = \omega_3 \cdot l_{O_3A_3}$	$V_D = \omega_9 \cdot l_{O_4-D}$	$V_S = \omega_9 \cdot l_{O_4-S}$	$V_N = \omega_{11} \cdot l_{O_5-N}$	$V_M = \omega_{11} \cdot l_{O_5-M}$
Velocity value, m/s	1.20	1.40	2.10	0.78	0.70	0.73	0.48

Table 3

Linear velocities of points of the third-class mechanism with three leading links

Designation of a part of the velocity plan	*Pb*	*Pc*	*Pe*	*Pf*
Value of the velocity plan part, mm	144.5	214.0	104.0	278.5
Linear velocity of a point	$V_B = Pb \cdot K_V$	$V_C = Pc \cdot K_V$	$V_E = Pe \cdot K_V$	$V_F = Pf \cdot K_V$
Linear velocity value of the point, m/s	4.19	6.21	3.02	8.08

ing links according to the velocity plan constructed on the scale $KV = 0.029$, (m/s)/mm are presented in Table 3.

Conclusions. In this work, a plan was developed, and a sequence of actions was implemented that allowed us to perform kinematic studies and determine the linear velocities of points coinciding with the geometric centres of rotating kinematic pairs and the absolute angular velocities of individual links of the complex planar mechanism of the third-class with three leading links by the graph-analytical method using the Autocad computer-aided design and drawing systems. The numerical values of the kinematic parameters were obtained and compared with the parameters of the same mechanism, which were calculated using the method of mathematical modelling in the Mathcad software environment. The coincidence of the results of research performed by two different methods with a 95 per cent probability was confirmed, which is a simultaneous confirmation of the reliability of the results obtained by such methods of analysis. If similar engineering calculations are obtained using mathematical modelling technologies, then expert studies of such complex mechanical systems of the highest class can be recommended to be carried out using the graph-analytical method.

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Визначення швидкостей точок механізму третього класу із трьома ведучими ланками графоаналітичним методом

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Мета. Розробка послідовностей дій і проведення кінематичного дослідження складного плаского механізму третього класу із трьома ведучими ланками графоаналітичним методом, визначення дійсних величин кутових швидкостей окремих ланок механізму та лінійних швидкостей усіх точок, що співпадають із центрами кінематичних пар механізму.

Методика. Кінематичне дослідження дванадцятиланкового механізму виконане з використанням положень курсу теорії механізмів і машин про теорію структурної будови механічних систем і кінематичного аналізу важільних механізмів графоаналітичним методом. Графічні побудови зроблені в системі автоматизованого проєктування та креслення Autocad, що дозволило точність графічних

побудов вивести на загальноприйнятий рівень проведення інженерних розрахунків.

Результати. Із використанням положень теорії будови механізмів вищих класів курсу теорії механізмів і машин, механізм із трьома ступенями рухомості було розглянуто у вигляді трьох механізмів, в яких послідовно досліджувався вплив руху однієї ведучої ланки з наперед заданими кінематичними параметрами на рух ланок механізму, для яких складали системи кінематичних рівнянь із подальшим їх розв'язуванням у вигляді графічних побудов. Отримано розрахунок дійсних кутових швидкостей тих ланок, абсолютний рух яких обумовлений їх кінематичним приєднанням до нерухомого стояка механізму. Виконано розрахунок лінійних швидкостей точок, що співпадають із геометричними центрами обертальних кінематичних пар ведених ланок механізму третього класу із трьома ведучими кривошипами.

Наукова новизна. Розроблено план і реалізована послідовність дій, яка дозволила виконати кінематичні дослідження й визначити лінійні швидкості точок, що співпадають із геометричними центрами обертальних кінематичних пар ланок складного дванадцятиланкового механізму третього класу із трьома ведучими ланками. Спеціально розроблена для такого плаского механізму із трьома кривошипами послідовність досліджень дозволила визначити дійсні значення кутових швидкостей ланок, абсолютний рух яких обумовлений їх кінематичним приєднанням до нерухомого корпусу. Це дозволило скласти системи векторних кінематичних рівнянь для визначення лінійних швидкостей точок механізму третього класу із трьома ведучими ланками та розв'язати їх у графічний спосіб.

Практична значимість. Отримані чисельні значення кінематичних параметрів і виконано їх порівняльний аналіз із параметрами цього ж механізму, що були розраховані за допомогою методу математичного моделювання у програмному середовищі Mathcad. Підтверджено збіг результатів досліджень, які виконані двома різними методами, із 95-ти відсотковою ймовірністю, що є одночасним підтвердженням достовірності результатів, отриманих такими методами аналізу. Рекомендовано проводити експертні дослідження складних механічних систем за допомогою графоаналітичного методу, якщо інженерні розрахунки отримані з використанням технологій математичного моделювання.

Ключові слова: *механізм третього класу, кінематичний аналіз, графоаналітичний метод, кінематичне дослідження*

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