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RESONANCE IN REAL RLC CIRCUITS AS THE INTERFERENCE OF HYPERBOLIC FREQUENCY WAVES

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1. Introduction. In [1, 5] precise formulas were obtained for the amplitude resonance frequency of a real (lossy) parallel RLC oscillatory circuit. In [2] was demonstrated how these results can be applied via duality to a real series RLC circuit. In this paper, we demonstrate that the symmetry, self-duality, and elegance of the resulting formulas are manifestations of the profound properties of hyperbolic waves in the frequency domain. These frequency waves are analogous to the forward and reverse waves in the time domain of a long line - the theory of the telegraph equation, which has wide practical applications.

Real series and parallel resonant circuits (Fig. 1) are special cases of a more general circuit - the RLC resonant bridge (Fig. 2). The study of the frequency characteristics of such a bridge was initiated in [3,4]. We use the notation and results of [3,4].

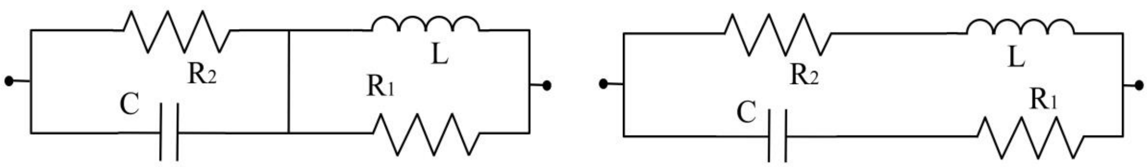

Figure 1 – Series (left, R_3= 0) and parallel (right,R_3=∞) circuits as special cases of the bridge circuit.

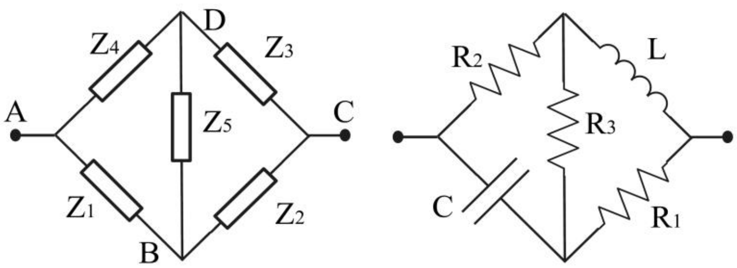

Figure 2 – A two-terminal network in the form of a bridge and its special case with two reactive elements

We use notations for the impedances of the bridge elements $Z_1 = \dfrac{1}{j\omega C}$, $Z_2 = R_1$, $Z_3 = j\omega L$, $Z_4 = R_2$, $Z_5 = R_3$, $x = \omega\sqrt{CL}$, $a = R_1\sqrt{C/L}$, $b = R_2\sqrt{C/L}$, $c = R_3\sqrt{C/L}$, that allow us to write the impedance of the bridge (Fig. 2) in this form

$$Z_{AB}=\sqrt{\frac{L}{C}}\cdot f(x,a,b,c)\,,\; f(x,a,b,c)=\frac{(a+b+c+abc)+j\big(a(b+c)x-b(a+c)/x\big)}{(1+ab+ac+bc)+j\big((b+c)x-(a+c)/x\big)} \tag{1}$$

For a series circuit c=0, and for a parallel circuit c=∞. Omitting the frequency-independent factor, we reduce the study of the frequency characteristics of the circuits in Fig. 1 to the study of these two functions

$$f_{ser}(x)=f(x,a,b,0)=\frac{(a+b)+jab\big(x-1/x\big)}{(1+ab)+j\big(bx-a/x\big)} \tag{2}$$

$$f_{paral}(x)=f(x,a,b,\infty)=\frac{(1+ab)+j\big(ax-b/x\big)}{(a+b)+j\big(x-1/x\big)} \tag{3}$$

Since the bridge in Fig. 2 is autodual in the sense of [2, 3, 4], then when replacing $a\Leftrightarrow 1/a$, $b\Leftrightarrow 1/b$, $c=0\Leftrightarrow 1/c=\infty$, we obtain

$$f\left(x,\frac{1}{a},\frac{1}{b},\frac{1}{c}\right)=\frac{1}{f(x,a,b,c)}\,,\qquad f_{ser}\left(x,\frac{1}{a},\frac{1}{b}\right)=\frac{1}{f_{parall}(x,a,b)} \tag{4}$$

2. Hyperbolic representation. The study of the amplitude-frequency characteristics of the circuits in Fig. 1 comes down to the study of the square of the impedance or admittance modulus

$$\left|f_{ser}(x)\right|^2=ab\frac{\left(\dfrac{a+b}{ab}\right)^2+\big(x-1/x\big)^2}{\left(\dfrac{1+ab}{\sqrt{ab}}\right)^2+\left(\sqrt{\dfrac{b}{a}}x-\sqrt{\dfrac{a}{b}}\dfrac{1}{x}\right)^2};\left|f_{paral}(x)\right|^2=ab\frac{\left(\dfrac{1+ab}{\sqrt{ab}}\right)^2+\left(\sqrt{\dfrac{a}{b}}x-\sqrt{\dfrac{b}{a}}\dfrac{1}{x}\right)^2}{\big(a+b\big)^2+\big(x-1/x\big)^2} \tag{5}$$

Let us move to logarithmic space by making the following substitution of variables and parameters in the circuit: $x=\exp(t)\,,a=\exp(\alpha)\,,b=\exp(\beta)$.

$$\left(\frac{1+ab}{2\sqrt{ab}}\right)^2=\cosh^2\left(\frac{\alpha+\beta}{2}\right),\frac{1}{4}\left(\sqrt{\frac{a}{b}}x-\sqrt{\frac{b}{a}}\frac{1}{x}\right)^2=\sinh^2\left(t+\frac{\alpha-\beta}{2}\right) \tag{6}$$

$$\left|f_{ser}(x)\right|^2=ab\frac{\big((a+b)/(2ab)\big)^2+\sinh^2(t)}{\cosh(t-\alpha)\cosh(t+\beta)},\quad \left|f_{paral}(x)\right|^2=ab\frac{\cosh(t+\alpha)\cosh(t-\beta)}{\big((a+b)/2\big)^2+\sinh^2(t)} \tag{7}$$

Thus, apart from constant, frequency-independent factors, the analysis of the square of the amplitude-frequency response of the circuits shown in Fig. 1 boils down to the study of two functions:

$$F(t,\alpha,\beta)=\frac{\left(\dfrac{a+b}{2ab}\right)^2+\sinh^2(t)}{\cosh(t-\alpha)\cosh(t+\beta)},\quad G(t,\alpha,\beta)=\frac{\left(\dfrac{a+b}{2}\right)^2+\sinh^2(t)}{\cosh(t+\alpha)\cosh(t-\beta)} \tag{8}$$

One of the functions corresponds to the impedance of the circuit, and the second to the admittance.The main thesis of this work is the fact that these functions can be represented as

$$F(t,\alpha,\beta)=R_s+P_s\tanh(t-\alpha)-Q_s\tanh(t+\beta) \tag{9}$$

$$G(t,\alpha,\beta)=R_p+P_p\tanh(t+\alpha)-Q_p\tanh(t-\beta) \tag{10}$$

The coefficients R_s and R_p can be found by sending t to plus and minus infinity. Furthermore, these limits give the difference of the other coefficients.

$$R_s=R_p=\cosh(\alpha-\beta)\,,P_s-Q_s=\sinh(\alpha-\beta)\,,P_p-Q_p=\sinh(\beta-\alpha) \tag{11}$$

The coefficients P_s , Q_s , P_p , Q_p can be found using standard methods that are used in the expansion of ordinary fractional rational functions using residues of the original functions at their poles – the points $t = i\frac{\pi}{2} \pm \alpha, t = i\frac{\pi}{2} \pm \beta$.

$$P_p = \frac{((a+b)/2)^2 - \cosh^2(\alpha)}{\sinh(\alpha+\beta)}, Q_p = \frac{((a+b)/2)^2 - \cosh^2(\beta)}{\sinh(\alpha+\beta)} \quad (12)$$

$$P_s = \frac{\cosh^2(\alpha) - \left(\frac{a+b}{2ab}\right)^2}{\sinh(\alpha+\beta)}, Q_p = \frac{\cosh^2(\beta) - \left(\frac{a+b}{2ab}\right)^2}{\sinh(\alpha+\beta)} \quad (13)$$

It can be shown that the coefficients in the expansions (9) and (10) is always positive.

3. Amplitude resonance. Expansion (9), (10) allows one to easily find the extreme point of the squared impedance (admittance) modulus by differentiating with respect to the variable *t*. The root of the derivative yields the amplitude resonance frequency of the circuit. Depending on whether this is a maximum or a minimum, we obtain a voltage resonance or a current resonance. To find the resonant frequency, we obtain the following equations:

$$\frac{\cosh(t+\alpha)}{\cosh(t-\beta)} = \sqrt{\frac{P_p}{Q_p}}, \frac{\cosh(t-\alpha)}{\cosh(t+\beta)} = \sqrt{\frac{P_s}{Q_s}} \quad (14)$$

The solutions of equations (14) will be as follows:

$$x_p^2 = \exp(2t) = \frac{b}{a} * \frac{ab\sqrt{P_p} - \sqrt{Q_p}}{ab\sqrt{Q_p} - \sqrt{P_p}}, x_s^2 = \exp(2t) = \frac{a}{b} * \frac{ab\sqrt{Q_s} - \sqrt{P_s}}{ab\sqrt{P_s} - \sqrt{Q_s}} \quad (15)$$

The conditions for the positivity of the expressions (15) provide the necessary and sufficient conditions for the existence of amplitude resonance of the circuits. These conditions exactly coincide (are special cases) with the conditions for the existence of resonance of the general bridge circuit shown in Fig. 2, which were found in [3,4]. In the initial variables *a*,*b*, the coefficients in the expansion (9), (10) take the form:

$$P_p = \frac{b(1+ab+2a^2)}{2a(1+ab)}, Q_p = \frac{a(1+ab+2b^2)}{2b(1+ab)} \quad (16)$$

$$P_s = \frac{\frac{1}{b}\left(1+\frac{1}{ab}+\frac{2}{a^2}\right)}{2\frac{1}{a}\left(1+\frac{1}{ab}\right)} = \frac{a+2b+a^2b}{2b(1+ab)}, Q_p = P_s = \frac{\frac{1}{a}\left(1+\frac{1}{ab}+\frac{2}{b^2}\right)}{2\frac{1}{b}\left(1+\frac{1}{ab}\right)} = \frac{b+2a+ab^2}{2a(1+ab)} \quad (17)$$

Substituting the formulas for the coefficients (16) and (17) into the root formulas (15), we obtain the values of the squares of the reduced amplitude resonance frequencies for the circuits shown in Fig. 1, which were first derived by a different method in [5,1]:

$$x_{paral}(a,b)^2 = \frac{\sqrt{1+ab+2b^2} - b^2\sqrt{1+ab+2a^2}}{\sqrt{1+ab+2a^2} - a^2\sqrt{1+ab+2b^2}} \quad (18)$$

$$x_{ser}(a,b)^2 = \frac{\sqrt{1+\frac{1}{ab}+\frac{2}{b^2}} - \frac{1}{b^2}\sqrt{1+\frac{1}{ab}+\frac{2}{a^2}}}{\sqrt{1+\frac{1}{ab}+\frac{2}{a^2}} - \frac{1}{a^2}\sqrt{1+\frac{1}{ab}+\frac{2}{b^2}}} == \frac{a\sqrt{a}}{b\sqrt{b}} \frac{\left(b\sqrt{ab}\sqrt{ab^2+2a+b} - \sqrt{a^2b+2b+a}\right)}{\left(a\sqrt{ab}\sqrt{a^2b+2b+a} - \sqrt{ab^2+2a+b}\right)} \quad (19)$$

3. Conclusion and Discussion. This study establishes a structurally transparent representation of the amplitude–frequency characteristics of linear lumped two-terminal networks. The key result is that the squared magnitude of the impedance (or admittance) can be expressed as the sum of a frequency-independent constant and two hyperbolic tangent functions with shifted arguments and distinct amplitudes (9),(10). This representation reveals a deep analogy with the theory of long transmission lines. In that context, voltage and current are described as superpositions of forward and backward traveling waves in the time domain. In the present work, the two hyperbolic tangent terms play an analogous role in the frequency domain: each term can be interpreted as a "frequency-domain wave" characterized by its own shift and amplitude.

At the same time, an essential distinction must be emphasized. In transmission line theory, waves propagate in time and space and satisfy hyperbolic partial differential equations. Here, the representation is purely frequency-based: the hyperbolic functions describe the redistribution of energy across frequencies rather than physical propagation.

Nevertheless, the analogy is conceptually fruitful. The interaction of the two hyperbolic components determines the resonance behavior, much like the interference of forward and backward waves determines standing-wave patterns in transmission lines. In summary, the proposed representation provides both analytical convenience and a new physical interpretation, linking lumped circuit theory with wave phenomena in distributed systems.

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