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APPLICATION OF GRAPH THEORY METHODS FOR SOLVING MECHATRONIC TASKS

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As it has been shown in [1] the task of multilayer printed boards development so as determining the optimal trajectory of the thread in the textile machine can be considered as mechatronic tasks. Both of tasks are solved with using the methods of finding an optimal path in the graph that is the model of the given construction. Since for separate cases using of general model causes exponential complexity of solving the task it is of some interest development the methods of decreasing the methods complexity that is the subject of given work.

In general, the problem of finding the optimal path in the graph is represented as follows. There is given undirected graph $G = (X, E)$, in which there are identified two vertices: s - initial vertex and t – final vertex. On the set of paths P of graph G a monotonic function $F(p)$, $p \in P$, is defined which creates single-valued transformation P in the set of real numbers. It is demanded to find $(s\ t)$–path $p^*(s, t)$, $p^* \in P$, where the function F reaches the minimum value (such (s t)-path is called optimal); similarly determined the optimal path connecting an arbitrary pair of vertices $x, y \in X$. In given case, the function F is monotony means that the function F on any way p accepts value no less than on any subpath $p^* \subseteq p$. From that it follows directly that in solving the problem we can be limited to consideration of only simple ways.

Proposed in [2] classification of target functions helps identify conditions which allow to use polynomial algorithm Dijkstra. For cases where the application of this algorithm does not let to obtain solution of the problem in [2] there is proposed a general scheme for solving that has exponential asymptotic computational complexity.

At the same time, [3] in solving the problem of determining the optimal trajectory thread it is shown that the objective function belongs to the class γ-functions and according to [2] requires the use of a common scheme for finding the optimal path in the graph relatively monotonous objective function. Moreover, in [3] there is proposed the transition from the original model as a graph, whose vertices correspond mutually uniquely to points of thread path direction changes, to another graph (auxiliary graph). The usefulness of such transition is determined by the solution of the original problem can be obtained with using of Dijkstra's algorithm to the auxiliary graph. In this case due to increasing of the initial graph vertices number it is achieved possibility to use polynomial algorithm instead of exponential one. The number of the auxiliary graph vertices is $O(n^2)$, where n - the number of vertices of the original model.

In the submitted work the question of transition from the initial graph to some auxiliary one for the use polynomial algorithm for finding the optimal path is solved. The analysis of the problem has established that an essential feature in solving problem submitted is dependence of objective function F increment when adding an edge (x, y) to the (s x)-path from the vertices preceding the vertex y. Namely, let (s x)-path has the form $p\ (s, x) = (s, x_1, x_2, ..., x_{k-r}, x_{k-r+1}, ..., x_{k-1}, x_k)$, where $x_k = x$ and (s y) -path is presented with sequence of vertices $p\ (s, y) = (s, x_1, x_2, ..., x_{k-r}, x_{k-r+1}, ..., x_{k-1}, x_k, y)$. Then function F increment in after addition the (s x)-path with edge (x, y) can be represented as $\Delta F = F\ (p\ (s, y)) - F\ (p\ (s, x)) = f\ (x_{k-r}, x_{k-r+1}, ..., x_{k-1}, x_k, y)$ where f - some real function.

If $r = 0$, it is said the increment of the objective function is independent of the background process past history, function F belongs to the class of α-functions and the Dijkstra's algorithm can be used. If $r > 0$, the auxiliary graph $H\ (V, U)$ is constructed as follows. Between the vertices V and sequences of $r + 1$ *vertices* $(x_1, x_2, ..., x_{r+1})$ such that $(x_i, x_{i+1}) \in R$, i = 1, ..., r, is stated one to one correspondence. Two vertices $v_1 = (x_1, x_2, ..., x_{r+1})$ and $v_2 = (x_2, x_3, ..., x_{r+2})$ are connected by an edge $(v_1, v_2) \in U$ if and only if $(x_{r+1}, x_{r+2}) \in R$. Since in $H\ (V, U)$ transition from any vertex $v_i \in V$ adjacent to v_j completely determines the appropriate increment of function F in finding the shortest way it is possible to use the Dijkstra's algorithm.

When determining the optimal trajectory thread $r = 1$, so vertices of the graph $H\ (V, U)$ correspond to two elements subsets of vertices of G, i.e. its edges. In general, the number of vertices H is the number of placements from n elements by $r + 1$: $A_n^{r+1} = n\ (n-1)\ ...\ (n-r)$. As $A_n^{r+1} = O\ (n^r)$ application of the Dijkstra's algorithm to graph H has polynomial asymptotic complexity.

To determine the optimal path for given orthogonal graph [2], when $r = 1$, we get a polynomial algorithm. As for the graph with colored vertices, the considered approach does not allow to improve exponential complexity algorithm as in the worst case $r = n$. But taking into account that in practice the number of vertex colors is considerably less then n it can be used the method of search path in the state space graph.

Let the number of vertex colors is equal to $k << n$. Then the number of different vertex colors for any path is limited with 2^k. That's why the number of vertices in the state space graph is equal to $n2^k$. And since value of 2^k is limited with a constant the search of the shortest path in the state space graph has a polynomial asymptotic estimate relatively vertices number.

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